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Optimality Of Inventory Decisions In A Joint Two-Echelon Inventory System With And Without Fuzzy Demand

[*]Vineet Mittal, [1]Dega Nagaraju and [2]S. Narayanan

[*]Final Year B.Tech. ComputerScience and Engineering,
School of Computing Science and Engineering, VIT University,
Vellore-632014, Tamil Nadu, India.
E-mail: vineetmittal04@gmail.com
[1]School of Mechanical and Building Sciences,VIT University, Vellore-632014, Tamil Nadu, India
E-mail: deganagaraju@vit.ac.in
[2]Pro-Vice Chancellor,VIT University, Vellore-632014, Tamil Nadu, India
E-mail: provc.vlr@vit.ac.in

Abstract

Inventory planning and control decisions are an important aspect of the management of many organizations. Inventory decision-making is the cognitive process of selecting a course of action from multiple alternatives. Fuzzy set approaches to decision-making are usually most appropriate for effective inventory control decisions. Industrial engineering brings a significant number of applications of fuzzy set theory. This paper considers a joint two-echelon inventory system with a single-vendor supplying a single type of product to a single-buyer. A mathematical model is developed by considering setup/ordering cost and carrying cost at respective entities of the supply chain. The objective of the proposed model is to demonstrate the optimality of inventory decisions and shipment frequency under non-fuzzy and fuzzy demand. A step-by-step algorithm is developed and a computer program is written in MATLAB to solve the model. Also, the sensitivity analysis is carried out to analyse the variation in model parameters over the optimality of decision variables and objective function.

Keywords: Supply chain;Non-fuzzy and fuzzy demand;Fuzzy set theory;Inventory control decisions;

1. Introduction

Supply chain management presents significant opportunities for improving margins and reducing cost. Inventory is a major source of cost in a supply chain and has a very large impact on responsiveness. Three groups of costs are associated with inventory[2]. The first group isinventory ordering costs whichconsists of costs associated with the acquisition of inventory like clerical costs and transportation costs. The second group isinventory carrying costs which consists of costs that arise from having inventory on hand like storage space costs, handling costs, property taxes, insurance, obsolescence losses and interest on capital invested in inventory. The third group iscosts of not carrying sufficient inventory,consists of costs that result from not having enough inventory on hand to meet customers' needs. Costs in this group are more difficult to identify than costs in the other two groups, but nevertheless they can include items that are very significant to a firm. Examples of costs in this group are customer ill will, quantity discounts foregone, erratic production (expediting of goods, extra setup, etc.), and inefficiency of production runs, added transportation charges and lost sales. Minimization of these costs is important to optimize the inventory model. Swarup et al. (2004), Panneerselvan (2006), Hira et al. (2007), and Taha (2009) have introduced various concepts of operations research which involve inventory management and supply chain coordination between manufacturer and retailer. They emphasized on the importance of reducing the various costs in the supply chain to optimize the profits and output.

Tu et al. (2011) stated the beneficial pricing strategy can benefit the vendor more than multiple buyers in the integrated system, when price reduction is incorporated to entice the buyers to accept the minimum total cost.Fuzzy set theory developed by Zadeh (1965) is an excellent conceptual and algorithmic framework to solve these problems.Xu et al. (2008) emphasized on the importance of use of the fuzzy numbers in supply chain coordination in a two-stage supply chain. The fuzzy demand is beneficial to optimize the supply chain. They prove that the maximum expected supply chain profit in a coordination situation is greater than the total profit in a non-coordination situation. In supply chain coordination, decision making is very important. Editorial (2007) says that decision-making is the cognitive process of selecting a course of action from multiple alternatives. Fuzzy set approaches to decision-making are usually most appropriate when human evaluations and themodeling of human knowledge are needed. Industrial engineering brings a significant number of applications of fuzzy set theory.

Yang (2006)proposed the inclusion of fuzzy annual demand and then employed the signed distance to find the estimation of the common total cost in the fuzzy sense in the two echelon inventory model. Two-echelon inventory optimization looks at inventory levels holistically across the supply chain while taking into account the impact of inventories at any given level or echelon on other echelons. For example, if the product sold in a retailer's outlet is received from one of its distribution centres, the distribution centre represents one echelon of the supply chain and the outlet another one. It should be clear that the amount of stock needed at the outlets is a function of the service received from the distribution centre. The better the service that is provided upstream, the smaller the protection that is needed downstream. The goal of two-echelon inventory optimization is to continually update and optimize safety stock levels across all of these echelons. Two-echelon inventory optimization represents the state of the art approach to optimize inventory across the end to end supply chain.Two-echelon inventory optimization simultaneously decreases the inventory levels and maintains high availability of the product for the customer.

In this paper, two-echelon inventory system is modelled under non-fuzzy and fuzzy demand model. A mathematical model is developed and corresponding numerical example is presentedalong with results and discussions. Sensitivity analysis for the dataset is done as well. Finally, conclusions and references are presented.

2. Literature Review

A brief review of literature pertaining to supply chain coordination is presented in this section. Several studies proposed joint inventory optimization models and addressed supply chain coordination in terms of lot size inventory, total relevant costs, etc. Many shipment policies were proposed in literature for two echelon inventory problem.

Kumar et al. (2014) demonstrated the optimality of the decision variables and objective function for respective entities as well as for the entire chain under exponential price dependent demand. From their research findings, it was evident that with supply chain coordination, the total relevant cost of supply chain decreases. Petrovic et al. (1999)developed a fuzzy model to determine the order quantities for each inventory in the supply chain in the presence of uncertainties that gave an acceptable service level of the supply chain at reasonable total cost.Cadenas et al. (2000) presented vector optimization problems with a fuzzy nature. They presented solution methodologies for the multiobjective fuzzy mathematical programming problems, using different ordering methods, ranking fuzzy numbers. Following this, Giannoccaro et al. (2003) presented methodology to define a supply chain inventory management policy, which is based on the concept of echelon stock and fuzzy set theory. The methodology was applied on a three stage supply chain so as to show the ease of implementation.Das et al. (2004) presented multi-item stochastic and fuzzy-stochastic inventory models formulated under total budgetary and space constraints. They solved a problem with single objective by a gradient-based technique and applied fuzzy technique to the multi-objective one.

Xiong et al. (2006) developed a fuzzy inventory model to counteract the demand fluctuation in supply demand networks, which combines fuzzy logic controller with policy based on economic order quantity model. The performance of the simulation was presented and discussed, which showed that the proposed multi-echelon fuzzy inventory model provides not only a cost-effective management of inventory in market uncertainty, but also another effective alternative for counteracting demand fluctuation. They proposed a multi-echelon fuzzy inventory model which showed benefit in counteracting demand fluctuation in multi-echelon supply demand networks.Editorial (2007) presented the editorial information of some research papers which further developed the area of fuzzy set applications in industrial engineering.Geetharamaniet al. (2007) applied fuzzy set theory to multi-echelon inventory system. They modulated and simulated the behaviour of a supply chain in an uncertain environment. Their paper establishes the study of this paper.

Zhou et al. (2008) considered a supply chain to be operated in a fuzzy environment. They developed expected value models as well as chance-constrained programming models to determine the pricing strategies for the retailer and the manufacturer. Then, Xu et al. (2008) developed an optimal technique for dealing with the fuzziness aspect of demand uncertainties. They used triangular fuzzy numbers to model external demand, and decision models in both non coordination and coordination situations were constructed. The behaviors and relationships of both the manufacturer and the retailer were quantitatively analyzed, and a cooperative policy for the optimization of the whole supply chain was put forward. Their work is the basis of the numerical illustration used in this paper. Xu et al. (2010) considered a two-stage supply chain coordination problem and focused on the fuzziness aspect of demand uncertainty. They used fuzzy numbers to depict customer demand, and investigate the optimization of the vertically integrated two-stage supply chain under perfect coordination and contrast with the non-coordination case. They proved that the maximum expected supply chain profit in a coordination situation is greater than the total profit in a non-coordination situation.

Tu et al. (2011) developed a two-echelon inventory model with mutual beneficial pricing strategy with considering fuzzy annual demand; single vendor and multiple buyers. Their paper proved that the price reduction mechanism is a mutual beneficial strategic partnership between the vendor and buyers.Yaghin et al. (2012) aimed to maximize the total profit of manufacturer, the total profit of retailer and improving service aspects of retailing simultaneously. They applied appropriate strategies to de-fuzzify the original model and then the equivalent multi-objective crisp model is solved by a fuzzy goal programming method. They provided an illustrative example to show the applicability and usefulness of the proposed model and solution method.Chang et al. (2013) investigated the effects of the manufacturer’s refund on retailer’s unsold products for the two-echelon decentralized and centralized supply chains of a short life and returnable product with trapezoidal fuzzy demand, in which retailer returns the unsold and the customer’s unsatisfactory products to the manufacturer. They provided a number of managerial insights by comparing both chains and show that each chain is more advantageous to the members depending on certain condition.

In light of the above literature, in this paper a two-echelon inventory model is developed under the decision variables to calculate the effect on total relevant costs in two different approaches that is without fuzzy and with fuzzy. Numerical illustration is shown with the help of MATLAB to understand the process. Then, sensitivity analysis is carried out to analyse the dataset and conclusions are put forward.

3. Mathematical Model Development

3.1 Assumptions

The proposed model in this study is developed on the following assumptions:

a) Infinite production rate
b) Replenishment rate is instantaneous
c) Manufacturer's inventory is multiple times of retailer's inventory level
d) Shortages are not allowed

3.2 Notations

ψ_R Total Relevant Cost of Retailer (in Rs.)

ψ_m Total Relevant Cost of Manufacturer (in Rs.)

ψ_S Total Relevant Cost of Supply Chain (in Rs.)

D Annual Demand rate of the product at the Retailer (in units)

α Left spread of triangular fuzzy demand $\tilde{D}$ (number of units)

β Right spread of triangular fuzzy demand $\tilde{D}$ (number of units)

A_R Ordering cost per order at the retailer (in Rs.)

C_R Procurement cost of the product for retailer (in Rs.)

I Carrying charge(in %)

A_m Setup cost per setup at the manufacturer (in Rs.)

C_m Unit cost at the manufacturer (in Rs.)

q_R Retailer's ordering quantity (in units) (Decision variable)

λ Number of shipments delivered from manufacturer to retailer (integer) (Decision variable)

q_m Manufacturer's replenishment batch size (in units)

3.3 Model Formulation without Fuzzy Demand

In the proposed mathematical model, two-echelon inventory system with single manufacturer supplying a single kind of product to a single retailer is considered. The model is developed to demonstrate the optimality of inventory decisions by considering the total relevant cost (ψ) of manufacturer, retailer as well as the supply chain.The annual total relevant cost of the supply chain is the sum of annual total relevant cost of the retailer and manufacturer[8].

The annual total relevant cost of the retailer is

$$\psi_R = \frac{DA_R}{q_R} + \frac{q_R C_R I}{2} \tag{1}$$

The annual total relevant cost of the manufacturer is

$$\psi_m = \frac{DA_m}{q_m} + \frac{q_m C_m I}{2} \tag{2}$$

$$\psi_m = \frac{DA_m}{\lambda q_R} + \frac{(\lambda - 1) q_R C_m I}{2} \qquad (q_m = \lambda q_R) \tag{3}$$

The annual total relevant cost of the supply chain is the sum of the annual total relevant costs of manufacturer and retailer.

$$\psi_S = \frac{D}{q_R}(A_R + A_m/\lambda) + \frac{q_R I}{2}(C_R + (\lambda - 1) C_m) \tag{4}$$

The annual total relevant cost of the supply chain expressed in equation (4) is strictly said to be convex in terms of retailer's ordering quantity, q_R and shipment frequency, λ.

Optimal ordering quantity of the retailer is obtained by taking the first order and second order partial derivatives of equation (4) with respect to q_R. Hence,

$$\frac{\partial(\psi_S)}{\partial q_R} = -\frac{D(A_R + A_m/\lambda)}{q_R{}^2} + \frac{I(C_R + (\lambda - 1) C_m)}{2} \tag{5}$$

$$\frac{\partial(\psi_S)}{\partial q_R} = 0 \tag{6}$$

$$q_R = \sqrt{\frac{2D}{I}\left(\frac{A_R + A_m/\lambda}{C_R + (\lambda - 1) C_m}\right)} \tag{7}$$

Next, carrying the second order partial derivative of equation (4) with respect to q_R,

$$\frac{\partial^2(\psi_S)}{\partial q_R^2} = \frac{2D(A_R + A_m/\lambda)}{q_R{}^3} \tag{8}$$

$$\because \frac{\partial^2(\psi_S)}{\partial q_R^2} > 0$$

for all values of λ, q_R and other model parameters. Hence, $q_R^* = q_R$.

Now, substituting the expression of q_R, in the original expression of ψ_S and with further simplification and rearranging the terms, the expression representing the total relevant cost of the supply chain is obtained as

$$\psi_S = \sqrt{2DI(A_R + A_m/\lambda)(C_R + (\lambda - 1) C_m)} \tag{9}$$

From equation (9), λ associated terms are separated and the following expression is obtained for the optimality of number of shipments.

$$F(\lambda) = (A_R + A_m/\lambda)(C_R + (\lambda - 1)C_m) \tag{10}$$

Now,

$$\frac{\partial(F(\lambda))}{\partial\lambda} = (A_R + A_m/\lambda)(C_m) + (C_R + (\lambda-1)C_M)\left(-\frac{A_m}{\lambda^2}\right) = 0 \tag{11}$$

$$\therefore \lambda = \sqrt{\frac{A_m(C_R - C_m)}{C_m A_R}} \tag{12}$$

As λ has to be an integer, we take the lower and upper integer values and compare the values of corresponding functions of λ. The function which has the smaller value gives the optimal value of λ.

3.4 Model Formulation with Fuzzy Demand

In this model, the demand factor is fuzzified and how that affects the decision variables in the model is seen.As, it is expressed by Xu et al. [20], afuzzified demand, $\tilde{D}$ is said to be triangular, if its membership function is expressed as shown below:

$$\tilde{D}(x) = \begin{cases} x/\alpha - (d-\alpha)/\alpha & \text{if } d-\alpha < x \le d \\ -x/\beta + (d+\beta)/\beta & \text{if } d < x \le d+\beta \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where d is the most possible value of fuzzified demand $\tilde{D}$ and α, β are the left and right spread of $\tilde{D}$ respectively. Hence, the fuzzified demand is denoted as $\tilde{D}$ =(d , α , β). If $\alpha = \beta$, $\tilde{D}$ becomes symmetric, and it can be written as $\tilde{D}$ =(d , α). It can easily be shown that the δ -level set of $\tilde{D}$, defined as $D_\delta = \{x \mid \tilde{D}(x) \ge \delta\}$, is a closed bounded interval for $0 \le \delta \le 1$, and can be denoted as

$$D_\delta = \left[d + (\delta - 1)\alpha, d - (\delta - 1)\beta\right] \tag{14}$$

When demand is fuzzy, then objective function becomes fuzzy. The probabilistic mean of fuzzy number $\tilde{D}$, introduced by Dubois and Prade [3], is defined as the average of the interval-valued expectation, namely

$$E(\tilde{D}) = \frac{E_*(\tilde{D}) + E^*(\tilde{D})}{2} \tag{15}$$

where $\left[E_*(\tilde{D}), E^*(\tilde{D})\right]$ is the interval-valued expectation. A more complete discussion of the probabilistic mean of a fuzzy number is presented in Dubois and Prade [3]. The advantage of this probabilistic mean is that it synthetically considers the information on every membership degree and its meaning is very intuitive and natural. Furthermore, we can verify that

$$E(\tilde{D}) = \int_0^1 \left(\frac{d_1(\delta) + d_2(\delta)}{2} \right) d\delta \tag{16}$$

where $[d_1(\delta), d_2(\delta)]$ is the δ –level set of $\tilde{D}$. It is easy to see that if $\tilde{D}=(d\ ,\alpha\ ,\beta\ )$ is a triangular fuzzy number, then

$$E(\tilde{D}) = \int_0^1 \left(\frac{d_1(\delta) + d_2(\delta)}{2} \right) d\delta = \int_0^1 \left(\frac{d + (\delta-1)\alpha + d - (\delta-1)\beta}{2} \right) d\delta = d + \frac{(\beta-\alpha)}{4} \tag{17}$$

$$\therefore \tilde{\psi}_S = \frac{(d+(\beta-\alpha)/4)\left(A_R + A_m/\lambda\right)}{q_R} + \frac{q_R I\left(C_R + (\lambda-1)C_m\right)}{2} \tag{18}$$

Now,

$$\frac{\partial\left(\tilde{\psi}_S\right)}{\partial q_R} = -\frac{(d+(\beta-\alpha)/4)\left(A_R + A_m/\lambda\right)}{q_R^{\;2}} + \frac{I\left(C_R + (\lambda-1)C_m\right)}{2} = 0 \tag{19}$$

$$\Rightarrow q_R = \sqrt{\frac{2\left(d+(\beta-\alpha)/4\right)\left(A_R + A_m/\lambda\right)}{I\left(C_R + (\lambda-1)C_m\right)}} \tag{20}$$

Now,

$$\frac{\partial^2\left(\tilde{\psi}_S\right)}{\partial q_R^2} = \frac{2(d+(\beta-\alpha)/4)\left(A_R + A_m/\lambda\right)}{q_R^{\;3}} > 0 \tag{21}$$

for all values of $q_{R,}\lambda$ and other model parameters. Hence,

$$\therefore q_R^* = q_R \tag{22}$$

Now, substituting the q_R value obtained in the original equation of $\tilde{\psi}_S$ to get the λ associated terms i.e. $F(\lambda)$ and then differentiating it to calculate λ^*,

$$\tilde{\psi}_S = \sqrt{2(d+(\beta-\alpha)/4)I\left(A_R + A_m/\lambda\right)\left(C_R + (\lambda-1)C_m\right)} \quad (23)$$

$$\therefore F(\lambda) = \left(A_R + A_m/\lambda\right)\left(C_R + (\lambda-1)C_m\right) \quad (24)$$

Now,

$$\frac{\partial\left(F(\lambda)\right)}{\partial\lambda} = 0 \quad (25)$$

$$\therefore \lambda = \sqrt{\frac{A_m\left(C_R - C_m\right)}{C_m A_R}} \quad (26)$$

Thus, in the fuzzy model, the optimal values of decision variables q_R and λ are obtained. Thesedecision variables under fuzzy demand will result in different value of total relevant costs associated with it, in comparison to non-fuzzy model.

3.4 Step by Step Algorithm

The following algorithm is developed to determine the optimal values of the two decision variables λ and q_R:

Step 1 Set the values of $D, I, \alpha, \beta, A_R, A_m, C_R$ and C_m.

Step 2 Determine λ using eq. (12).

Step 3 If λ is integer proceed to Step 5, otherwise Step 4.

Step 4 Find $F(\lambda)$ for upper (λ_1) and lower (λ_2) integers from eq. (10)

(a) If $F(\lambda_1) < F(\lambda_2)$ then $\lambda^* = \lambda_1$

(b) Else $F(\lambda_2) < F(\lambda_1)$ then $\lambda^* = \lambda_2$.Go to Step 6.

Step 5 $\lambda^* = \lambda$

Step 6 Determine q_R using eq. (20) and q_m using eq. (3)

Step 7 Determine ψ_R using eq. (1), ψ_m using eq. (3) and ψ_S using eq. (4)

4. Numerical Illustration

For the illustration of the model, consider an item with the following variables and constant values. The model is solved using a MATLAB program written based on the step by step algorithm for the data shown below and the results are tabulated in Table1.

D = 5000 units per year I = 18% p.a.
α = 3500 β = 6500
A_R = Rs. 150 per order A_m = Rs. 450 per setup
C_R = Rs. 140 per unit C_m = Rs. 100 per unit

Table 1: Optimal Values of Decision Variables and Total Relevant Costs

Parameter	Without Fuzzy Demand	With Fuzzy Demand
q_R^* (in units, decision variable)	487.95	523.26
λ^* (a +ve integer, decision variable)	1	1
q_m^* (in units)	487.95	523.26
ψ_R^* (in rupees)	7685.21	8241.47
ψ_m^* (in rupees)	4611.12	4944.88
ψ_S^* (in rupees)	12296.34	13186.35

Table 1 shows the optimal values of retailer's replenishment quantity, number of shipments, manufacturer's replenishment batch size and total relevant costs of respective entities as well as the supply chain, with and without fuzzy demand. From table 1, it is evident that retailer's replenishment quantity is more with fuzzy demand rather than without fuzzy demand. However, there is no change in the number of shipments for both the cases. Similarly, under fuzzy demand, the optimal values of manufacturer's replenishment batch size, total relevant costs of the retailer, manufacturer and the supply chain are higher compared to non-fuzzy demand. It is attributed to the fact that the values obtained in both cases, it is observed that the proportionate increase in the quantity takes place which in turn increases the costs of the supply chain but not by the same proportion. The decision variables of the inventory affect the costs when demand is non fuzzy and fuzzy. Thus, in the fuzzy model, the supply chain delivers a higher optimal ordering quantity, which optimizes the total relevant costs.

Table 2: Sensitivity Analysis

Parameter	(in %)	Without Fuzzy Demand						With Fuzzy Demand					
		q_R^*	λ^*	q_m^*	ψ_R^*	ψ_m^*	ψ_S^*	q_R^*	λ^*	q_m^*	ψ_R^*	ψ_m^*	ψ_S^*
A_m	+ 40	556.3	1	556.3	8358	5661.9	14019.9	596.6	1	596.6	8963.0	6071.7	15034.7
	+ 20	523.2	1	523.2	8026.4	5159.8	13186.3	561.1	1	561.1	8607.4	5533.3	14140.7
	- 20	449.8	1	449.8	7335.4	4001.1	11336.6	482.4	1	482.4	7866.4	4290.7	12157.2
	- 40	408.2	1	408.2	6981.0	3306.8	10287.8	437.7	1	437.7	7486.3	3546.1	11032.4
A_R	+ 40	511.7	1	511.7	8499.9	4396.5	12896.5	548.8	1	548.8	9115.2	4714.7	13829.9
	+ 20	500.0	1	500.0	8100.0	4500.0	12600.0	536.1	1	536.1	8686.2	4825.7	13511.9
	- 20	475.5	1	475.5	7254.0	4730.9	11984.9	510.0	1	510.0	7779.1	5073.3	12852.4
	- 40	270.0	2	540.0	5068.8	6596.4	11665.3	289.5	2	579.1	5435.7	7073.9	12509.6
C_m	+ 40	487.9	1	487.9	7685.2	4611.1	12296.3	523.2	1	523.2	8241.4	4944.8	13186.3
	+ 20	487.9	1	487.9	7685.2	4611.1	12296.3	523.2	1	523.2	8241.4	4944.8	13186.3
	- 20	307.7	2	615.4	6314.5	5871.4	12186.0	330.0	2	660.0	6771.6	6296.4	13068.0
	- 40	322.7	2	645.4	6390.4	5228.5	11618.9	346.1	2	692.2	6852.9	5606.9	12459.9
C_R	+ 40	265.2	2	530.5	7506.8	6628.1	14135.0	284.5	2	569.0	8050.2	7107.9	15158.1
	+ 20	278.8	2	557.6	6905.6	6544.2	13449.9	298.9	2	597.9	7405.4	7017.9	14423.4
	- 20	545.5	1	545.5	6873.8	4124.3	10998.1	585.0	1	585.0	7371.3	4422.8	11794.2
	- 40	-	-	-	-	-	-	-	-	-	-	-	-
I	+ 40	412.3	1	412.3	9093.2	5455.9	14549.2	442.2	1	442.2	9751.4	5850.8	15602.3
	+ 20	445.4	1	445.4	8418.7	5051.2	13469.9	477.6	1	477.6	9028	5416.8	14444.9
	- 20	545.5	1	545.5	6873.8	4124.3	10998.1	585.0	1	585.0	7371.3	4422.8	11794.2
	- 40	629.9	1	629.9	5952.9	3571.7	9524.7	675.5	1	675.5	6383.8	3830.2	10214.1
D	+ 40	577.3	1	577.3	9093.2	5455.9	14549.2	607.4	1	607.4	9568.0	5740.8	15308.8
	+ 20	534.5	1	534.5	8418.7	5051.2	13469.9	566.9	1	566.9	8929.4	5357.6	14287.0
	- 20	436.4	1	436.4	6873.8	4124.3	10998.1	475.5	1	475.5	7490.6	4494.3	11984.9
	- 40	377.9	1	377.9	5952.9	3571.7	9524.7	422.5	1	422.5	6655.5	3993.3	10648.9

Further, the sensitivity analysis is carried out for the above numerical example to understand the variations in the optimality of decision variables and total relevant costs to analyse the effect of inventory decisions in the two-echelon inventory system. From Table 2, it is observed that the variation in the model parameters is influencing the optimality of decision variables and objective function for both the cases of fuzzy and non-fuzzy demand. Also from Table 2, it is found that when the retailer's ordering cost is reduced by 40%, there is an increase in the value of number of shipments which shows that retailer can increase the frequency of orders by reducing its ordering cost.The reduction in manufacturer's unit cost leads to the increase in the number of shipments. When the manufacturer's unit cost is increased by 20% and 40%, the optimal values of decision variables and objective function remain same for both the cases. Numbers of shipment are increased, when the retailer's unit cost is increased by 20% and 40% under both models. So, to increase the shipments frequency, one can increase the retailer procurement cost. Similarly, when the value of interest rate is reduced by 20% and 40%, the replenishment quantities increased whereas the total relevant costs decreased. Thus, the data in the sensitivity analysis follows the general trend of optimization and shows how different variables behave under different variations. The data under thefuzzy demand shows an increase in quantity and proportionate increase in the costs making the system optimal as compared to the data under non-fuzzy demand.

5. Conclusions

This paper mainly dealt with the development of a quantitative model for two echelon inventory system for optimal total relevant cost of the supply chain, the manufacturer and the retailer and the decision variables of the respective entities under the influence of non-fuzzy and fuzzy demand. The model presents a comparative study of decision variables for both models ofsupply chain. From the research findings of the model, it is concluded that when the demand factor is fuzzified, the corresponding ordering quantity increases proportionally, which in turn increases the total relevant costs of retailer and manufacturer. Also, increases the total relevant cost of the supply chain but the quantity levels of the chain are optimal and beneficial for both echelons.The manufacturer produces more and retailer receives more, increasing their total relevant costs and making the supply chain more optimal overall. From the sensitivity analysis it is also concluded that the total relevant cost of the supply chain increases with increase in ordering cost, set up cost and unit cost at the retailer and manufacturer.Thus, the current work helps the management in taking optimal inventory decisions and shipment frequencies under fuzzy and non-fuzzy demand in a joint two-echelon inventory system.

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