Master's Thesis

How The U.S. Government Finances its Deficits: Macroeconomic Effects of Government Debt Management *

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Abstract

Global financial market turmoil, leading to abrupt increases in fiscal deficits have been experienced by many countries during the recent downturn, bringing to light concerns about government debt sustainability. This thesis analyzes fiscal policy, debt management and the dynamic adjustment of government debt in response to economic shocks. This is done with an evaluation of how alternative ways of managing the maturity structure of debt affects the dynamic behavior of the government's liability within the context of general equilibrium theory. We utilize a standard macroeconomic framework in which a household optimizes and the government issues debt in short and long term bonds. To calibrate the model to the empirical observations, we estimate the debt management rule from the data. With our economic model we consider the effects of the current debt management regime followed by authorities in the US and contrast with alternative rules which are common in the literature. We show that in the case where the government issues only short maturity debt and in the case where it buys back its debt in every period, there are significant effects on the dynamics of the market value to GDP ratio, however there are more moderate effects on consumption and hours. To motivate these findings we appeal to the theory of fiscal insurance, which suggests that long maturity government bonds have a hedging value to the intertemporal budget. We conclude that, to the extend that governments are concerned about high debt levels, the choice of the debt management regime is important as it affects the dynamics of the debt aggregate.

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1 Introduction

The current turmoil in global financial markets and the abrupt increase in fiscal deficits experienced by many countries during the recent downturn has brought to surface concerns about the sustainability of government debt. These concerns are especially relevant for several countries in the EU, but also for the US where the rise in the debt level has been unprecedented in the post World War II era and has led to heated political debate concerning acceptable debt levels.

It is therefore evident that analyzing fiscal policy, debt management and understanding the dynamic adjustment of government debt in response to economic shocks is a matter of primary importance. This thesis attempts to do so by evaluating how alternative ways of managing the maturity structure of debt affect the dynamic behavior of the government's liability within the context of general equilibrium theory. To realize this goal we take the following steps. First, we look closely at the data to discern the current debt management practice in the US, that is we investigate what maturities debt is issued in and in what proportions. Second, we embed debt management in a macroeconomic model which is broadly similar to the models used by Angeletos (2002), Marcet and Scott (2009) and Faraglia et al. (2014 (b)) to study optimal fiscal policy jointly with debt management.

To be more specific, our structural model is an economy with a representative household and a government. The economy is hit by spending and technology shocks which may drive the governments budget to deficit. In order to finance the deficit, the government levies distortionary taxes to the household's labor income and issues debt in two maturities: one year and ten year bonds. The household optimizes so that bond prices are given by the familiar Euler equations (i.e. the ones which equate the price of a bond of maturity N to the expected growth rate in the marginal utility of consumption between periods t (today) and $t + N$ years, times the appropriate rate at which the household discounts the future consumption flows. Moreover, in contrast to the literature on debt management which typically assumes that taxes are set optimally by a government which maximizes the households welfare, we take a process of labor income tax rates which, according to the literature, approximates well fiscal policy in the US. We further assume that all debt in the economy is real.

Given our assumptions on the maturity structure (and which are typical in the literature), we attempt to map our model to the data. In particular, we utilize the CRSP database to answer the following question: 'If we assume that the debt management authority in the US issues debt in one and ten year bonds, how is the issuance strategy impacted by interest rates, by the level of government debt outstanding and by the volume of debt issued in the previous year?'. For this purpose we estimate the rule via which the debt management authority chooses in any given year, the fraction of one year debt over the total issuance. Our results indicate a strong relationship between the current share of one year debt, its first order lag, and the debt to GDP ratio outstanding. In contrast interest rates do not seem to impact the share.

At the heart of our analysis is the notion that governments may finance spending shocks either through current and future increases in tax rates, or through changes in the bond prices which impact the value of the outstanding debt obligation. In Section 2 of this thesis we explain this principle (to which we refer, following the related literature, as fiscal insurance) in detail. The basic intuition is the following: If rises in spending (or more generally shocks which lead to budget deficits) are accompanied by drops in bond prices, then governments may benefit by holding their debt in maturities whose prices are most sensitive to the shocks. Based on this principle Angeletos (2002) illustrated that if a government wishes to gain from fiscal insurance then the optimal debt management strategy is to issue only long term government debt. This conclusion is also reached by Buera and Nicolini (2006) and Faraglia et al. (2010) under different model specifications.

In section 3 we turn to the US data to ask whether long term debt issuances are a characteristic of the US debt management strategy. We find that whilst a fraction of government debt is indeed issued in long maturity, another considerable fraction is issued in short (one year) maturity. We conclude that at least in the historical observations the debt management authorities in the US have not sought to benefit from fiscal insurance.

Given these observations, and with our estimates of the issuance strategy described previously, we utilize our structural model (in Sections 4 and 5) to investigate how debt management may impact the economy by considering the historical policy but also simple alternative rules of managing the maturity structure. Our analysis focuses on two basic reforms in debt management and studies their effects on the dynamics of the market value of government debt, on tax rates and on private sector consumption and hours.

The first change in policy we consider has to do with simplifying the debt issuance strategy. In particular, we eliminate the dependence of the share of short term debt on the debt to GDP ratio, keeping all of our other estimates constant. We illustrate that this change effectively eliminates all long term debt from the economy so that the government finances deficits exclusively through one year bonds. We consider this change also for the following reason: In macroeconomic models with government debt it is typical to assume that all debt is basically of one model period maturity. We therefore wish to illustrate that ignoring the more complex debt management strategy employed by the US government may have effects on the dynamics of the government's liability, the tax outcomes and the overall implications of the model. For example, we find that the debt aggregate displays more volatility and larger swings in response to government deficit shocks. We attribute the increased volatility to a loss in terms of fiscal insurance that the government incurs through issuing only short maturity debt.

The second change in policy we consider relates to an institutional feature of debt management which is also largely underexplored in economic models. In particular it has been well documented in the empirical literature (see Marchesi (2004)) that governments across OECD countries (and hence also in the US) do not buy back their debt before it matures. Indeed in the US it is rather rare to observe large buybacks of long term non-maturing debt with the notable exception of the quantitative easing policies in 2009 and the buyback policies in 2001 (see for example Greenwood and Vayanos (2010)). Our baseline model is built on the assumption of no buy back. Also, our estimates of the debt management strategy followed, build on that assumption.

In contrast to the common case in practice, economic models which consider long maturity bonds (as for example the models of Angeletos (2002) and Faraglia et al (2010)) typically assume that once the government issues debt in these bonds, it removes this debt from the market one period after the issuance. Using our benchmark model which features no-buyback and comparing with a version of the model where we force the government to buyback its outstanding debt in every period we show that this dimension of debt management significantly affects the way the macroeconomy responds to changes in the deficit.

To the best of our knowledge several points made by this thesis are new to the literature. First, the explicit estimation of the sharing rule (done in section 3) is of our own design. In order to make our derivations (which rely on the assumption that the government issues one and ten year debt under no buyback) tractable and easy to map to the data, we have to employ a linear approximation. This enables us to drop the history of the past issuances and shares of short and long term bonds, and to summarize this history with the lagged average maturity structure (which we compute from the data). We therefore provide a tractable formula that is suitable to map the debt management policies found in Angeletos (2002), Buera and Nicolini (2006) and Faraglia et al. (2014 (b)) to the US data.

Second as discussed, this study is the first to propose integrating debt management in a dynamic stochastic general equilibrium model without studying the optimal fiscal policy problem (as is done in Angeletos (2002) Faraglia et al. (2014 (b)). As we said previously our approach is to summarize the US institutions in a simple law of motion for the labor income tax rate, which we take from previous estimates in the literature. In this sense our intention is not to investigate how the optimal tax schedule changes when, say, we give to the government the option to buyback its debt (as in Faraglia et al. (2014 (b)). Rather, we wish to see how the behavior of the economy is impacted when the course of fiscal policy is held fixed and we change the debt management practice.

The rest of this thesis is organized as follows: The next section presents the related literature and discusses the principle of fiscal insurance. Section 3 looks at the broad empirical facts on debt management in the US and also contains our estimates of the debt management rule. Section 4 presents the theoretical framework, the calibration of the model and the numerical algorithm which is utilized to solve it. Section 5 presents our quantitative results. A final section concludes.

2 Financing spending shocks through taxes, inflation and bond returns

In this section we describe the principle of financing government deficits through bond returns (what we referred to in the introduction as fiscal insurance). For this purpose we utilize in our analysis the government's budget constraint as a key object (following Aiyagari et al. (2002) and Angeletos (2002)). In order to simplify we assume that spending shocks are the only source of variation in the government budget. As in most of the related literature, we take spending as exogenous and assume that its value fluctuates according to a first order autoregressive stochastic process.[1] Because our theoretical model in section 4 assumes that all government debt is real we start by describing the results in the literature which relate to how an active debt management of the portfolio of real government debt, can help achieve fiscal sustainability. Subsequently, and for the sake of making our overview more complete, we will briefly describe how inflation can also be utilized to the same effect.

2.1 The case of real government debt

Consider the dynamics of government debt in an economy where debt is real. Let g_t denote the value of the total expenditures of the government sector in period t and let τ_t denote the tax rate levied. Assume for simplicity that the tax base is total output, denoted by Y_t, which in turn is produced by a technology that features labor as the sole input in production. [2]

In order to make the point that taxes are distortionary, assume that we can write $Y_t = Y(\tau_t, g_t)$ with the property $Y_1 < 0$). Moreover, note that the above characterization of output (as a function of τ_t, g_t) is not complete as a characterization of the equilibrium in the economy. In a complete characterization, tax rates themselves should be a function

[1] This assumption is rather common in dynamic general equilibrium macro models (i.e. that spending can be represented as a first order autoregressive process and is completely exogenous to the economic system).

[2] This assumption is rather common in the literature. For the purpose of the exposition we will maintain the notation Y_t for aggregate output here and leave it to section 4 where we present our full macroeconomic model to replace output with hours.

of g_t and the level of government debt outstanding. In order to simplify the exposition we adopt this notation here.

Let us also assume for the moment that g_t is the only shock to the economy (this assumption will be lifted in later sections). Moreover, let b_t^i be a bond of maturity $i = 1, 2...$ years that the government issues to finance its deficit. If we focus on the case $i = 1$ (one year maturity as in Aiyagari et al. (2002)) we can write the government's budget constraint as:

$$b_t^1 q_t^1 = b_{t-1}^1 + g_t - \tau_t Y_t \tag{1}$$

where b_{t-1}^1 is the debt issued in the previous quarter by the central government, $g_t - \tau_t Y_t$ is the primary surplus in t and q_t^1 is the bond price (inverse of the one year interest rate on debt). According to (1) the total deficit of the government (primary deficit + amount of maturing debt) is financed by new debt issued in period t. Notice that the price of the bond of maturity q_t^1 is an endogenous object determined in equilibrium by the investor's (bondholder's) preferences, their consumption, and ultimately as a function of the tax policy and the debt level. Moreover, note that since debt is of one quarter, the price of maturing debt b_{t-1}^1 (q_t^0) is by definition equal to one.

Equation (1) can be iterated forward to give us the intertemporal constraint of the government. This is a key object on which we base our analytical results and discussion in this section. Noting that:

$$b_{t+1}^1 q_{t+1}^1 = b_t^1 + g_{t+1} - \tau_{t+1} Y_{t+1} \tag{2}$$

and substituting (2) into (1) we get:

$$q_t^1 (b_{t+1}^1 q_{t+1}^1 - g_{t+1} + \tau_{t+1} Y_{t+1}) = b_{t-1}^1 + g_t - \tau_t Y_t \tag{3}$$

or

$$q_t^1 (b_{t+1}^1 q_{t+1}^1 - g_{t+1} + \tau_{t+1} Y_{t+1}) - g_t + \tau_t Y_t = b_{t-1}^1 \tag{4}$$

Continuing with similar substitutions for future periods we get:

$$\sum_{j=1}^{\infty} q_t^1 q_{t+1}^1 ... q_{t+j-1}^1 (-g_{t+j} + \tau_{t+j} Y_{t+j}) - g_t + \tau_t Y_t = b_{t-1}^1 \tag{5}$$

or

$$\sum_{j=1}^{\infty} \Pi_t^{t+j-1} q_k^1 (-g_{t+j} + \tau_{t+j} Y_{t+j}) - g_t + \tau_t Y_t = b_{t-1}^1 \tag{6}$$

where $\Pi_t^{t+j-1} x_k$ is the product of x from period t to period $t+j-1$.

Note that in (6) we assume, that the values of all future levels of government spending are fully predictable. For a more general treatment of the intertemporal budget (one which allows for stochastic realizations of spending) we have to introduce to expression (6) the conditional expectation operator in period t (E_t):

$$E_t \sum_{j=1}^{\infty} \Pi_{k=t}^{k=t+j-1} q_k^1 (-g_{t+j} + \tau_{t+j} Y_{t+j}) - g_t + \tau_t Y_t = b_{t-1}^1 \tag{7}$$

Equation (7) represents the intertemporal constraint of the government (see for example Faraglia et al. (2014 (b)). It basically states that given the outstanding liability b_{t-1}^1, the financing of the governments debt can be accomplished either through the adjustment in the sequence of primary surpluses: $s_{t+j} = -g_{t+j} + \tau_{t+j} Y_{t+j}$ or through a change in current and future bond returns $q_t^1 q_{t+1}^1 ... q_{t+j-1}^1$ which the government may be (partially) able to influence. For example, assume that $b_{t-1}^1 > 0$ (i.e. the government has debt outstanding). Then according to (7) it must be that in expectation the government runs a surplus in future periods. Also if a shock is realized and the value of g_t increases, the intertemporal budget can balance in two ways: First, with the adjustment of the sequence of taxes upwards (which increases the surplus value). Second, with the change in bond returns $q_t^1 q_{t+1}^1 ... q_{t+j-1}^1$. Since we assume that $s_{t+j} > 0$ it must be that bond prices increase (or interest rates decrease) to finance spending. If neither of these conditions are met (so that the left of (7) is greater than the right hand side) then government debt is not solvent.

2.1.1 Bond prices in equilibrium

In order to put further structure to this argument we will now substitute out bond prices. Assume that there is one household in the economy whose preferences are of the form $u(c_t)$ where c_t is the value of the households consumption in t. Moreover let $\beta < 1$ be the households discount factor, i.e. the relative importance the household attaches to future consumption relative to present consumption.

Standard results (see for example Aiyagari et al. (2002)) imply that with these assumptions, a bond of maturity i has a price: $q_t^i = \beta^i E_t \frac{u_c(t+i)}{u_c(t)}$.[3] With this property we can express the product $E_t q_t^1 q_{t+1}^1 ... q_{t+j-1}^1$ as: $E_t \beta \frac{u_c(t+1)}{u_c(t)} \beta \frac{u_c(t+2)}{u_c(t+1)} ... \beta \frac{u_c(t+i)}{u_c(t+i-1)}$ or equivalently as: $E_t \beta^i \frac{u_c(t+i)}{u_c(t)}$. [4]

With these derivations we can write equation (7) as follows:

(8)
$$E_t \sum_{j=1}^{\infty} \beta^j \frac{u_c(t+j)}{u_c(t)} (-g_{t+j} + \tau_{t+j} Y_{t+j}) - g_t + \tau_t Y_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)}{u_c(t)} (-g_{t+j} + \tau_{t+j} Y_{t+j}) = b_{t-1}^1$$

Note that (8) now represents the intertemporal constraint of the government borne out of the equilibrium (i.e. government debt is priced at the rate the investor is willing to pay to hold it). Moreover, we anticipate that the pricing term $\beta^j \frac{u_c(t+j)}{u_c(t)}$ could be influenced by the tax schedule and the value of government spending in t and $t + j$.

2.2 Long Bonds and Fiscal Insurance

We now elaborate on how a carefully chosen portfolio of different maturities can facilitate the financing of spending shocks. For this purpose we continue to work with the government's intertemporal budget, however, rather than using exclusively short term (one

[3] This property follows from the household's optimization and is basically the so called Euler equation of the household. On the one hand the cost of investing q_t^i dollars today measured in terms of marginal utility (foregone consumption) is given by $q_t^i u_c(t)$. On the other hand the future benefit of collecting in $t + i$ 1 dollar which is the real payout of the government bond in this case, is given by: $\beta^i E_t u_c(t + i)$. The household invests in a bond of maturity i to the point where the cost is equal to the benefit.

[4] To reach the above formula we use (several times) the law of iterated expectations: In particular, it holds that $E_t q_t^1 q_{t+1}^1 ... q_{t+j-1}^1$ is equal to $E_t \beta \frac{u_c(t+1)}{u_c(t)} E_{t+1} \beta \frac{u_c(t+2)}{u_c(t+1)} ... E_{t+i} \beta \frac{u_c(t+i)}{u_c(t+i-1)}$. Starting from the last term note that $E_{t+i-1} \beta \frac{u_c(t+i-1)}{u_c(t+i-2)} E_{t+i} \beta \frac{u_c(t+i)}{u_c(t+i-1)} = E_{t+i-1} \beta \frac{u_c(t+i-1)}{u_c(t+i-2)} \beta \frac{E_{t+i} u_c(t+i)}{u_c(t+i-1)} = E_{t+i-1} \beta^2 \frac{E_{t+i} u_c(t+i)}{u_c(t+i-2)} = \beta^2 \frac{E_{t+i-1} u_c(t+i)}{u_c(t+i-2)}$. We apply this reasoning to all the terms from $t + 1$ to $t + i$ to reach the expression in text.

period) debt we add a long maturity. The results described in this paragraph can be found in Angeletos (2002), Buera and Nicolini (2006), and Faraglia et al. (2010) among others.

Let us for simplicity assume that along with one year debt, the government issues debt in an N year bond. With this addition we can write the government's per period budget constraint as follows:

$$(9)\quad b_t^1 \beta^1 E_t \frac{u_c(t+1)}{u_c(t)} + b_t^N \beta^N E_t \frac{u_c(t+N)}{u_c(t)} = b_{t-1}^1 + \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} b_{t-1}^N + g_t - \tau_t Y_t$$

and following the arguments in Faraglia et al. (2014 (b) we can derive the intertemporal constraint as:

$$(10)\qquad E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)}{u_c(t)} (-g_{t+j} + \tau_{t+j} Y_{t+j}) = b_{t-1}^1 + \beta^{N-1} \frac{u_c(t+N-1)}{u_c(t)} b_{t-1}^N$$

Equation (10) is similar to (8) with the addition of the term $\beta^{N-1} \frac{u_c(t+N-1)}{u_c(t)} b_{t-1}^N$. This term shows that when the government issues long term debt, and in response to a fiscal shock, there is an extra margin of adjustment on the intertemporal budget. To see this, assume that $b_{t-1}^N > 0$ and consider, as previously, the case where g_t increases unexpectedly. Again in this case the present discounted value of the surplus and the sequence of prices $\beta^j \frac{u_c(t+j)}{u_c(t)}$ needs to adjust in order to balance the intertemporal budget. However, if the positive innovation to spending is associated with a reduction of the term $\beta^{N-1} \frac{u_c(t+N-1)}{u_c(t)}$, then the required adjustment of the left hand side of (10) is less, as the government experiences a capital gain in its debt portfolio.

This logic lies behind the so called principle of fiscal insurance. According to this principle, governments which seek to minimize the distortionary impact of taxation should issue long term debt, as government spending shocks (or more generally shocks that lead to budget deficits) also lead to increases in long term interest rates. Through the ensuing depreciation of long bond prices and the devaluation of government debt, the government can avoid having to increase taxes abruptly in order to finance its spending.

2.3 No Buyback

The previous paragraph revisited the argument that long term debt can be beneficial to tax smoothing. However, it derived the intertemporal constraint of the government under the assumption that in each period government debt is bought in, independent of maturity, and new debt of either long or short maturity is issued to replace it. To see this notice that in equation (9) the right hand side of the period budget features the term $b^1_{t-1} + \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} b^N_{t-1}$ which represents the total expenditure (principal plus interest rate) on government debt outstanding in t. If we further assume that $N = 10$ (i.e. that the long term bond is of ten year maturity), then (9) suggest that a bond of ten years issued in $t-1$ is redeemed in t as a nine $(N-1)$ year bond.

This assumption of buying back the debt each period is a persistent feature of economic models such as the ones used by Angeletos (2002) and Faraglia et al. (2010), but is counterfactual to assume in reality. In practice governments rarely buy back their outstanding debt before it matures, as revealed in Faraglia et al. (2014 (b)). Rather debt once issued is expected to be redeemed at maturity.

To make these arguments more concrete note that equation (10) was derived from forward iteration of the following period constraint:

$$b^1_t \beta^1 E_t \frac{u_c(t+1)}{u_c(t)} + b^N_t \beta^N E_t \frac{u_c(t+N)}{u_c(t)} = b^1_{t-1} + \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} b^N_{t-1} + g_t - \tau_t Y_t$$

However maintaining the assumption that there are only two maturities available to the market and the government does not buy back its debt every period it is more appropriate to write:

$$b^1_t E_t \beta^1 \frac{u_c(t+1)}{u_c(t)} + b^N_t E_t \beta^N \frac{u_c(t+N)}{u_c(t)} = b^1_{t-1} + b^N_{t-N} + g_t - \tau_t Y_t$$

whereby an N year bond issued N years ago, and which matures in t has a price equal to one. Note that the above equation assumes that long term government bonds pay out zero coupons. We will later illustrate how this equation generalizes if we assume that the government pays coupons of amount κ each period.

In order to illustrate how imposing to hold bonds to maturity impacts the governments finances we can derive the intertemporal budget constraint of the government as follows:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)}{u_c(t)}(-g_{t+j} + \tau_{t+j} Y_{t+j}) =$$

$$b^1_{t-1} + \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} b^N_{t-1} + \beta^{N-2} E_t \frac{u_c(t+N-2)}{u_c(t)} b^N_{t-2} + ... + b^N_{t-1} \tag{11}$$

The right hand side of (11) is different from that of (10) because now the entire history of issuances of long term bonds matters for the solvency condition. In contrast, in (10) there is only the bond issued in the previous quarter b^N_{t-1}.

Note that in this respect the intertemporal budgets in (10) and (11) are not equivalent. The fact that long term debt in (10) achieves fiscal insurance is in a way saying that the government's intertemporal budget is at least partly state contingent, so that tax increases are not excessive in response to fiscal shocks. However, whereas in (10) the fiscal insurance benefit comes from the term b^N_{t-1} in (11) it derives from b^N_{t-1}, b^N_{t-2},...b^N_{t-N+1} since these terms are also multiplied by endogenous bond prices.

Another way of saying this is the following: Assume that two governments follow the same strategy of issuing short term and long term debt. However, assume that the first government does not buy back its debt obligations and the second government does buy back. Under (10) and (11) it is straightforward to argue that in any given period t the two governments will have different maturity structures of their total debt obligation. And since the maturity structure of debt is the object which ultimately determines the gains from fiscal insurance, buying back or not government debt is an important feature of debt management. With our economic model in section 4 we look at precisely these effects.

One final comment is important: In the following section where we document the properties of US debt management using historical data, we work with the stocks of government debt. Since we are interested in understanding how the debt management strategy maps into the fiscal insurance properties of the portfolio, we treat a long term bond issued in the past as a bond of shorter maturity. For example in the following expression

$$(12) \qquad b^1_{t-1} + \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} b^N_{t-1} + \beta^{N-2} E_t \frac{u_c(t+N-2)}{u_c(t)} b^N_{t-2} + ... + b^N_{t-N}$$

we assume that one year term debt is represented by $b^1_{t-1} + b^N_{t-N}$ (i.e. the sum of all maturing debt), two year debt by $\beta E_t \frac{u_c(t+1)}{u_c(t)} b^N_{t-N+1}$ and so on. This mapping to the data is necessary because given the above expression in t it is straightforward to show that the fiscal insurance properties of debt management under no buyback, depend on the timing of the issuance. Effectively a long term bond issued a long time ago gives less insurance in the current period. We follow the same empirical strategy when we deal with non zero coupon bonds.

2.3.1 Non zero coupons

Assume now that instead of a long term bond which pays out a given amount at maturity as principal, the government issues debt in non zero coupon bonds. In particular let κ be a constant coupon paid each period and let the government pay the coupon amount in every period and the coupon plus the principal (here normalized to one) at maturity. Formally, a bond of maturity N promises the following stream of payments:

$$\underbrace{\kappa}_{\text{Year 1}} \quad \underbrace{\kappa}_{\text{Year 2}} \quad .... \quad \underbrace{1+\kappa}_{\text{Year } N}$$

Under these assumptions it is possible to show (as in Faraglia et al. (2014 (b))) that the price of the N maturity bond is given by:

$$q^N_t = \kappa \sum_{j=1}^{N} \beta^j \frac{u_{c,t+j}}{u_{c,t}} + \beta^N \frac{u_{c,t+N}}{u_{c,t}}$$

Moreover, generalizing the government's intertemporal budget to non zero coupons (i.e. following the same procedure of iterating forward on the per period constraint) we can write:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)}{u_c(t)} (-g_{t+j} + \tau_{t+j} Y_{t+j}) = \tag{13}$$

$$b_{t-1}^1 + \kappa \sum_{j=1}^{N} \beta^{N-j} \frac{u_c(t+N-j)}{u_c(t)} b_{t-1}^N + \beta^{N-1} \frac{u_c(t+N-1)}{u_c(t)} b_{t-1}^N + ... + + b_{t-N}^N (1+\kappa)$$

Equation (13) generalizes the fiscal insurance argument to the case of non-zero coupon bonds. It basically states that relative to the case of zero coupons (analyzed in the previous paragraph) when the government issues debt in coupons, the average maturity of debt becomes shorter. Hence the fiscal insurance properties of the governments portfolio differ in this case.

2.4 The role of inflation in achieving fiscal solvency

Thus far we have illustrated in the case of real debt how the government can finance spending shocks through bond returns on the optimal portfolio. It was shown that when the government issues long term debt (in large quantities (as in Angeletos (2002)) spending shocks lead to a drop in the market value of debt, and thus to a capital gain from the portfolio. This devaluation of debt enables the government to smooth the distortionary burden from taxes as the necessary adjustment of the fiscal surplus (which comes through a rise in taxes) is lesser.

In this section we argue that when the government can influence the course of prices and there are nominal bonds available to the market, there is a similar fiscal insurance benefit from inflation adjustments. In particular when a shock drives the budget into deficit, an unexpected increase in inflation can reduce the real payout of government debt and therefore reduce the debt level.

Note that though we intend in our subsequent analysis to follow through with the assumption that government debt is real we feel that if we were not to mention (even briefly) the role of inflation as an alternative policy tool, our review of the literature would be incomplete. We therefore briefly derive here the intertemporal constraints to the case where the inflation margin is present.

Let P_t be the price level in the economy and B_t^i be the current price value of a zero coupon bond of maturity i. If we assume (for simplicity) that $i = 1$ (or that the government can buy back its debt in every period) we can express the per period budget constraint as follows:

$$B_t^1 q_t^1 = B_{t-1}^1 + P_t g_t - \tau_t Y_t P_t$$

dividing by P_t we can write:

$$\frac{B_t^1}{P_t} q_t^1 = \frac{B_{t-1}^1}{P_t} + g_t - \tau_t Y_t$$

and letting $\frac{B_t^1}{P_t} = b_t^1$ (i.e. the real value of debt) we can rearrange it into:

$$b_t^1 q_t^1 = \frac{b_{t-1}^1}{\pi_t} + g_t - \tau_t Y_t$$

where π_t denotes the gross inflation rate between t and $t+1$. Moreover standard results (see for example Faraglia et al. (2013)) imply that $q_t^1 = \beta E_t \frac{u_c(t+1)}{u_c(t)\pi_{t+1}}$ since the bond price is that of a nominal bond.

If we iterate forward in this expression we will obtain the government's intertemporal budget as follows:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)P_t}{u_c(t)P_{t+j}}(-g_{t+j} + \tau_{t+j}Y_{t+j}) = \frac{b_{t-1}^1}{\pi_t} \tag{14}$$

Note that (14) makes clear that inflation can contribute towards reducing the value of government debt if needed. Suppose that g_t increases and that $b_{t-1}^1 > 0$. In such a case if there is a positive inflation shock (so that π_t rises) the real payout of government debt will drop. Moreover (14) can be generalized to long term debt as follows:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)P_t}{u_c(t)P_{t+j}}(-g_{t+j} + \tau_{t+j}Y_{t+j}) = \frac{b_{t-1}^N}{\pi_t} E_t \beta^{N-1} \frac{u_c(t+N-1)}{u_c(t)\pi_{t+1}...\pi_{t+N-1}} \tag{15}$$

It is evident from (15) that when government debt is long term it is not only inflation in t which contributes towards fiscal insurance and the sustainability of government debt, but also inflation in $t+1$, $t+2$ ..., $t+N-1$. Therefore long term debt confers an advantage in the sense that with a persistent shock in inflation there can be a bigger reduction in the value of debt outstanding as future inflation rates matter. Equivalently, with long term debt, the government can spread the cost of inflation over several periods when they need to engineer a drop in the value of outstanding debt. This point has been raised by Lustig et al. (2008) and Faraglia et al. (2013). It can also be generalized to no buyback and non zero coupon bonds. For the sake of brevity we omit the derivations.

2.4.1 Why Inflation is Left Out of the Model.

As discussed, the model we build in sections 4 and 5 is one of real government debt. In this respect the analysis which follows makes no use of the effects of changes in the price level to the real liability (and hence the real debt burden) of the government. One could perhaps think that changes in inflation (in the context of an economic model) act as a shock, which may increase or reduce the the right hand side of equation (15) and also exert an influence on the discount factor on the left hand side. Thus changes in debt management (which are considered in the model) could in principle interact in a non-trivial way with the monetary policy rule and in general with the US institutions in the money market.

Such additions to the analysis could add considerable complexity, since (as is well known) the monetary policy regime has changed in the historical observations, becoming more conservative and arduous in its efforts to contain inflation. Hence inflation levels have been larger in the 70s and the 80s (presumably due to the large swings in oil prices which occurred in that period) but were substantially reduced in the 90s and the 2000s. However, one needs to remember that insofar as inflation is anticipated and therefore priced in the nominal interest rate, it exerts little influence on the government's budget position. To put it differently, in such a case, inflation does not affect the real interest rate on government debt. Rather, as follows from the derivations above, it is the non-systematic (surprise) component of inflation which may impact the government's finances.

It turns out that the non-systematic component has not made a large contribution to debt sustainability in the US in the post World War II era as advocated by Hall and

Sargent (2010). Their analysis seems to suggest that given the US institutions, there is little to be gained from adding the interaction between monetary and fiscal policies to the model, or from separating between nominal and real debt in the empirical analysis. Moreover, as is shown in Smidt Grohe and Uribe (2004) and Faraglia et al. (2013) in the context of models which feature jointly optimal monetary and fiscal policies, the policy maker has only a very small incentive to engineer changes in inflation, to manage the debt level. Most of the adjustments (in these models) which occur on the government's intertemporal budget come from the real surplus side (e.g. from tax rates). Hence also from the theoretical side the role of inflation is not substantial.

For these reasons we think that adding inflation to the analysis is not of first order importance.

3 Broad Historical Facts of US Government Debt Management

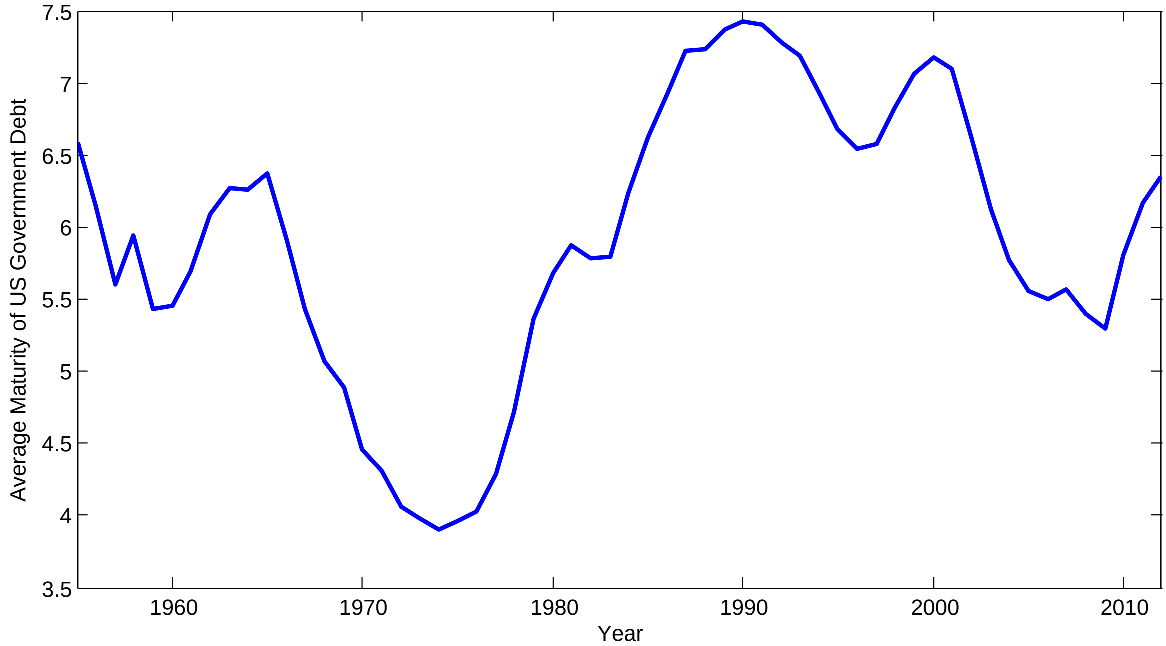
This section presents some broad facts on the debt management strategy of the US government in the period 1955-2011. The data are taken from the CRSP database and they correspond to issuances of debt in various maturities in the US. A detailed description of the variables utilized in this section can be found in the Appendix.

The US government issued debt in bonds of maturities ranging from one quarter to 30 years in the period studied. All bonds involve a payout at maturity and a sequence of coupon payments. Coupons are typically paid every six months. Moreover, the size of the coupon is such that the bond trades on average at par. This practically means that the government sets the coupon so that the price of a bond which promises to pay 100 dollars after say 10 years is today roughly equal to 100 dollars.

To study the government debt management practice closely we strip the coupons. We therefore, treat the sequence of payments promised by a long term bond of N quarters as a sequence of N bonds of maturity 1 to N. This calculation permits us to map the maturity structure of government debt to the fiscal insurance benefit that the government enjoys given its outstanding liability. As discussed in section 2, a bond of 10 year maturity with coupons does not provide the same insulation of the government's intertemporal budget as a ten year zero coupon bond (since the average maturity of debt is different in the two cases). In this sense it is important to characterize the outstanding debt maturity in order to make a meaningful connection between the debt management strategy we see in practice and the theoretical analysis presented in the previous section.

In Figure 1 we show the average maturity structure of the US government debt. As is indicated by the figure, over the sample period, the average maturity of debt changed considerably. In 1955 the US government issued debt of average maturity 6.5 years. In the 70s and the 80s the average maturity was roughly 4.3 and 6.5 years respectively, and finally more recently, in the 2000s it was 6 years. This evidence suggests that over the sample period there have been alterations of the debt management practice in the US. The changes in the maturity documented in Figure 1 are not driven by the business cycle;

Figure 1: **Average Maturity of Debt in the US**



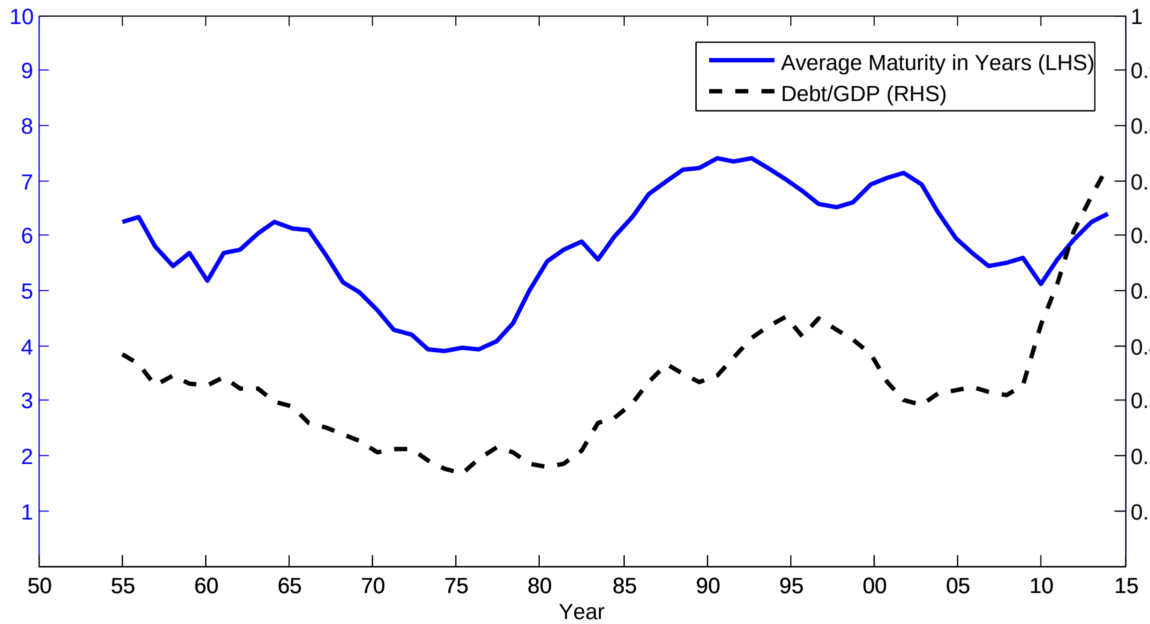
Notes: The Figure plots the average maturity of outstanding debt in the US over the period 1955-2011. The data are annual observations (time aggregated from monthly data extracted from the CRSP). Details on the data construction are contained in the Appendix.

rather these changes occur in the medium to long run. It is well known (see for example Greenwood et al. (2013)) that in periods where the overall debt level was high, the fiscal authorities have issued more long term debt, and vice versa, when debt was low there was a larger share of short bonds in the government's portfolio.

To further illustrate this point, in Figure 2, we plot the debt to GDP ratio in the US (on the right axis) and the average maturity structure (on the left) over the sample period. As is evident from the figure there is a strong positive correlation between the two aggregates. We calculate the correlation coefficient to be 0.56.

In order to decompose the evidence further, in Figure 3 we show the share of short term debt in the US, defined by convention as all debt that is maturing in less than one year, and in Figure 4 medium term and long term debt. Medium term debt includes all bonds of maturity between 2 and 5 years and long term debt more than 5 year maturities.

Figure 2: **Average Maturity of Debt in the US vs. Debt-to-GDP**

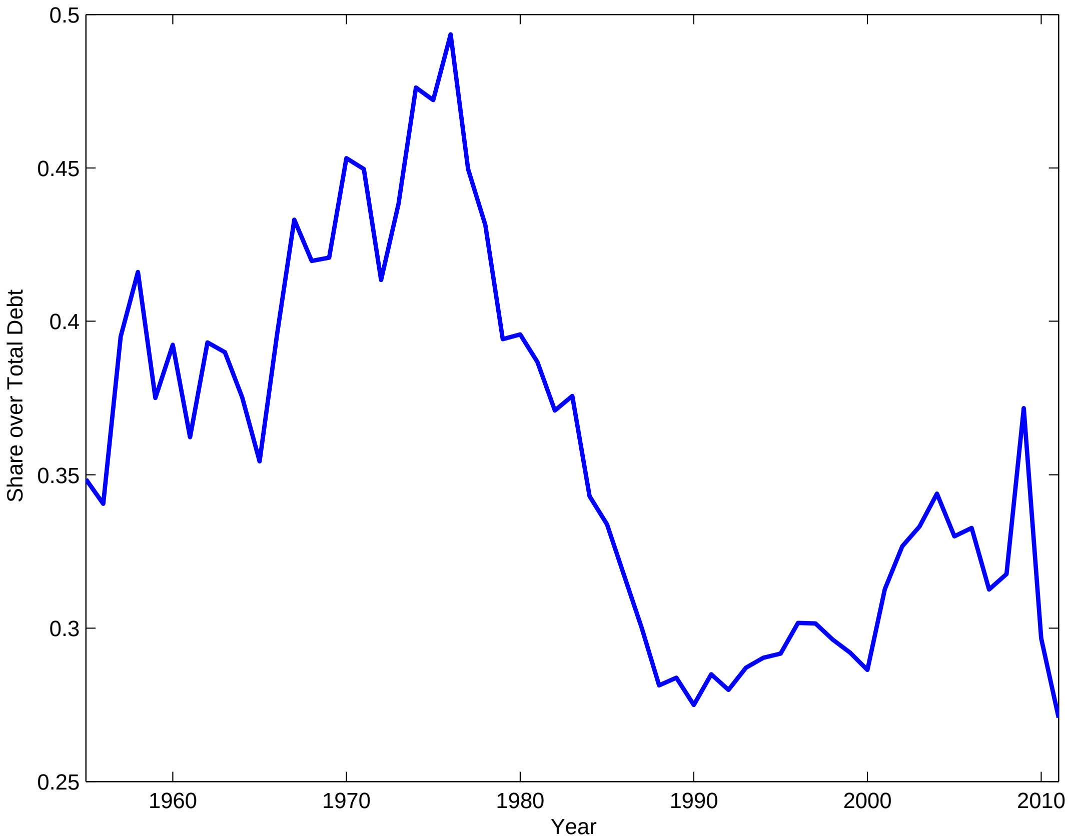


Notes: The Figure plots the average maturity of outstanding debt in the US against the total debt to GDP ratio over the period 1955-2011. The data are annual observations (time aggregated from monthly data extracted from the CRSP for the average maturity). The debt to GDP ratio was taken from the St Louis Fed's FRED database. Details on the data construction are contained in the Appendix.

There are several noteworthy features: First, government debt is held in both short, medium and long maturity. We calculate a substantial average share of short term debt (36% over the sample period). Second, there is a drop in average maturity in the 70s and the 80s which is partly driven by a drop in the share of short term debt and a rise in long maturity debt. As is evident from the graph, there is very little (if at all) variation in medium maturity debt. Rather, as the graphs suggest, all variations in the maturity structure are driven by substituting bonds at the short end of the term structure (essentially liabilities between 1 and 4 quarters) with bonds of very long maturity (say 10 years). [5]

[5] This finding is perhaps not surprising given the procedure we have utilized to partition government debt into different maturities. As explained above in order to better capture the fiscal insurance properties of debt, and also the opportunities that government debt gives to the private sector to transfer resources across periods, we have partitioned non zero coupon bonds into different maturities. Therefore, when

Figure 3: **Share of Short Term Debt in the US**

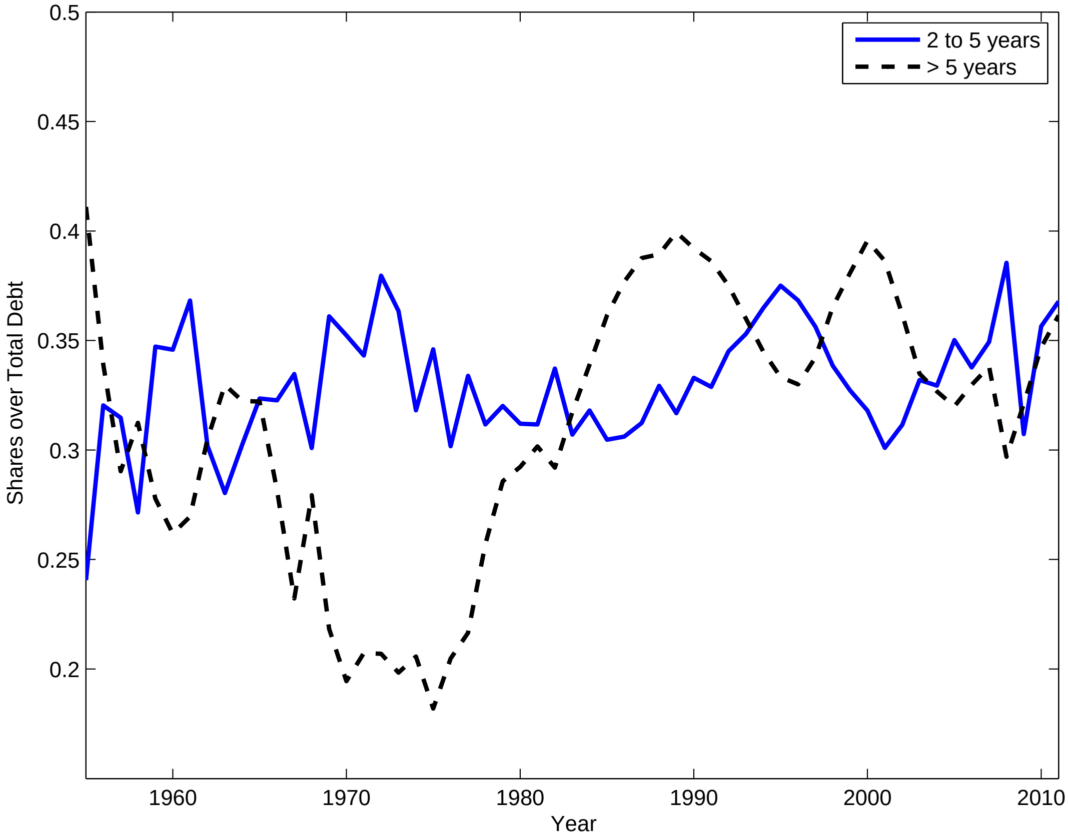


Notes: The Figure plots the share of short maturity government debt (less than or equal to one year) in the US over the period 1955-2011. The data are annual observations (time aggregated from monthly data extracted from the CRSP). Details on the data construction are contained in the Appendix.

It is worth noting that the above results have important implications for our modelling choices in the next sections. In order to capture parsimoniously the debt management practice in the US and to make our model tractable we will assume that the government issues debt in two maturities: a short term bond of one year maturity and a long term, ten year bond. These assumptions, though obviously simplistic, are essentially common in the literature (see Buera and Nicolini (2006), Faraglia et al. (2010, 2014 (b))). However,

a ten year bond is issued by the debt management office, its coupons contribute to shorter maturities, and as bond quantities of very long term debt issued in the past work their way through the maturity structure, they exert a similar influence. Therefore, in contrast to very short term debt (which is well known to be actively managed) and very long term debt, the quantities of intermediate bonds are less discernible given the way we construct the data. If this is indeed the case we would anticipate that a higher issuance of say ten year debt today will be compensated by a lower quantity in 5 year bonds issued after 5 years, consistent with the view that the government would target to have a stable quantity of middle term debt in the market. Though this is a possibility, we do not attempt to address it here. Rather we follow the rest of the literature (for example Hall and Sargent (2012)) in our definition the maturity structure of government debt.

Figure 4: **Breakdown of Debt in the US**



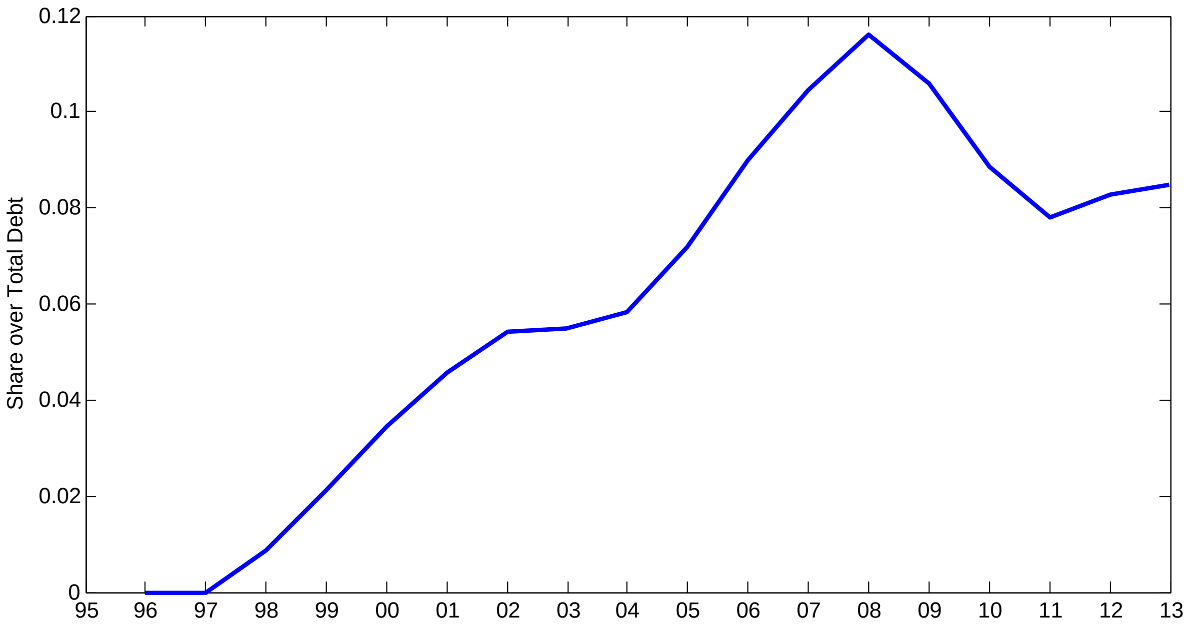
Notes: The Figure plots the share of government debt of different maturity buckets in the US over the period 1955-2011. The solid line represents the share of debt with a maturity greater than one year but less than five years (blue). The dashed line represents the share of debt with a maturity greater than five years (black). The data are annual observations (time aggregated from monthly data extracted from the CRSP). Details on the data construction are contained in the Appendix.

the results presented in this paragraph suggest that, by focusing on very short and very long maturities, we may nonetheless be able to accurately capture the changes in debt management that we see in the data.

3.1 Real and Nominal Debt

In Figure 5 we plot the share of indexed government debt over total debt in the US. The sample considered starts from 1995 and ends in 2012. The reason for omitting earlier years from the figure is that all US government debt, prior to 1997 was nominal. Starting in the late 90s the authorities issued progressively more indexed debt, which reached its maximum value (of roughly 12%) in 2008. Moreover, real debt is predominately long term in the US thus contributing to a longer maturity structure. [6]

Figure 5: **Share of TIPS in the US**



Notes: The Figure plots the share of real debt over total debt outstanding in the US over the period 1995-2011 (TIPS were introduced in 1997). The data are annual observations (time aggregated from monthly data extracted from the CRSP). Details on the data construction are contained in the Appendix.

As discussed previously, the analysis in the later sections draws no distinction between real and nominal debt. For that matter our economic model is based on the assumption that all debt is real or the equivalent assumption that inflation has only a minor impact on the dynamics of government debt. As we explained in section 2 this assumption is a good approximation of the historical data, and therefore a feature of US policy. However, for the sake of completeness, we discuss in this paragraph the historical observations on real bonds in the US.

[6] In essence, Treasury Inflation-Protected Securities (or TIPS as they are more commonly known), are assets which are primarily held by institutional investors such as pension funds. The reason is obviously that they provide insurance against the risk that long run (unexpected) inflation may substantially reduce the real payout of nominal assets. Households which are not willing to bear this risk (even though inflation risk premia could be substantial), wish to hold indexed debt in their portfolios.

3.2 Parameterization of the Debt Management Rule

For our quantitative model in section 5 we wish to discern the rule on which the debt management authority in the US decides its debt issuance. In this section we present our empirical strategy which estimates the share of short term one period debt over the total issuance. This rule which will give us the share as a function of economic fundamentals is new to the literature and we believe it can adequately map a model of two maturities (one year and ten year) to the debt management data.

3.2.1 Summarizing the Central Features of Debt Management in the US

The debt management practice has the following salient features:

1. Government debt issued is predominantly nominal.

2. Bonds are non zero coupon. In our CRSP dataset coupons are paid every six months and are chosen so that bonds prices trade close to (or at) par.

3. Debt is typically redeemed at maturity (no buybacks).

4. There is a large number of different maturities issued, ranging from one quarter to 30 years.

We have already discussed the relevant empirical evidence for points 1-4. 1. was shown in section 3 through studying the real bond issuances in the US historical observations. 2. and 3. are evidence for the US provided by Marchesi (2004) and also discussed in Faraglia et al. (2014 (b)). We also encountered these observations in the CRSP data set. As we explained in order to construct the maturity structure in the US and to make our analysis consistent with the notion that governments which issue more long term debt gain more from fiscal insurance, we had to strip the coupons, so that a long term (say ten year) bond is viewed as a sequence of different maturities (up to ten years). 4. was illustrated in section 3 through separating maturities into short term medium and long term, but is also a well known feature of debt management. We discuss extensively the properties of our data set in the Appendix.

Our parameterization of debt management does not precisely account for all of these facts. It builds a parsimonious model of issuances which, following the existing literature (for example Angeletos (2002) and Faraglia et al. (2014 (b)), splits the debt portfolio into short term and long term debt. Given that our model period is one year (see below), we set the short maturity to one year. Moreover, following the bulk of the literature we assume that long term debt is ten year debt. Given these observations we assume the ten year bond pays a coupon denoted by κ in every year. The government (in our benchmark analysis) is forced not to buy back this bond, until the bond matures. Finally, as discussed previously we draw no distinction between nominal and real debt since in our economic model in sections 4 and 5 we do not consider inflation as a policy margin.

3.2.2 The Debt Issuance Rule

Our characterization of debt management is a rule which governs the issuance of short term one year debt. With this rule we can uncover how the debt management office in the US finances the deficit (by generating revenue from short term and long term debt) and also construct the maturity structure of debt (given the sequence of issuances and the coupons) applying the same procedure we utilized to analyze the data (explained in further detail in the Appendix).

Let s_t^1 be the fraction of new debt issued in one year bonds in period t. We assume that s_t^1 is given by the following equation:

$$s_t^1 = \omega_1 + \rho s_{t-1}^1 + \omega_2 \frac{\text{Debt}_{t-1}}{\text{GDP}_{t-1}} + u_t \tag{16}$$

where ω_1 is a constant giving the intercept of the share, ρ is a parameter which measures the persistence, and ω_2 measures the response of the issuance to the debt to GDP ratio (lagged by a year). Finally u_t is a mean zero i.i.d. disturbance with constant variance (denoted by σ_u). u_t basically captures that the share s_t^1 contains a non-systematic (random) component.

As we have previously shown the share of short term debt in the US data responds strongly to variable $\frac{\text{Debt}_{t-1}}{\text{GDP}_{t-1}}$. We have illustrated that in periods where the debt level was high, the share of short bonds dropped (giving a strong positive correlation between the debt level and the average maturity of debt). Therefore, it is important to consider $\frac{\text{Debt}_{t-1}}{\text{GDP}_{t-1}}$ as a potentially significant variable. [7] In contrast, (16) does not acknowledge an influence of the term structure of interest rates to s_t^1. For example, it may be intuitive to think that when the cost of long term debt is high relative to the interest cost of short debt (i.e. the yield curve is steeply sloping upwards) then a cost minimizing debt management authority will respond by issuing more short term debt. However, cost minimization is not a primary objective of the US authorities it seems; as we have illustrated, changes in the share of short bonds occur mostly in the medium and the long run and do not exhibit a discernible business cycle pattern (as term spreads do). We have indeed verified that interest rates are not significant to include in (16). To keep the analysis focused on the significant variables we therefore omit them from the text.

3.2.3 Structural Estimation of the Sharing Rule

Note that even though equation (16) in principle can be run with the OLS using the data observations on s_t^1, the estimated parameters are likely to lead to a very biased debt management rule. This is the case because (as discussed) the US debt management office does not issue only one year and 10 year bonds, but rather issues debt in many different maturities. In this respect by directly estimating (16) from the data, key moments such as the maturity structure would be, in the model, very inaccurate approximations of their data counterparts.

Our strategy is to estimate a rule that matches the time series on the maturity structure which we observe in the data. In particular, we can show that given our assumptions on the issuances and the bonds which are available to the market, the maturity structure

[7] It is worth noting that what was defined as short term debt in the US economy to construct the observations for Figure 3 differs from the definition of s_t^1. In particular the share in Figure 3 was based on stripping the coupons and counting all bonds which have in any period one year of outstanding maturity as short term debt. s_t^1 is a measure defined over the issuance in a given year and hence refers exclusively to one year maturity debt (and not long term debt which is close to maturity). Nevertheless, the relation between s_t^1 and $\frac{\text{Debt}_{t-1}}{\text{GDP}_{t-1}}$ remains.

takes the following form: [8]

$$
(17) \qquad MAT_t = \frac{1}{MV_t}(b_t^N(p_t^N N + \sum_{j\in\{1,2,...N\}} jp_t^j\kappa_t))
$$

$$
+b_{t-1}^N(p_t^{N-1}(N-1) + \sum_{j\in\{1,2,...N-1\}} jp_t^j\kappa_{t-1}) + ... + b_{t-N+1}^N p_t^1(1+\kappa_{t-N+1}) + b_t^1 p_t^1)
$$

where $N = 10$ denotes the duration of the long term bond, MV_t is the market value of government debt, κ_t is the coupon paid on the long term bond issued in t and p_t^j represents the bond price of maturity j in t. [9]

The above equation can be further rearranged into:

$$
(18) \qquad MAT_t = s_t^1\omega_t + (1-s_t^1)\omega_t\xi_t +
$$

$$
\frac{MV_{t-1}}{MV_t}(1+i_{t-1}^1)\frac{b_{t-1}^N(Np_{t-1}^N\frac{p_t^{N-1}p_{t-1}^1}{p_{t-1}^N} + \sum_{j\in\{1,2,...N\}} jp_{t-1}^j\frac{p_t^{j-1}p_{t-1}^1}{p_{t-1}^j}\kappa_{t-1}) + ... + b_{t-1}^1 p_{t-1}^1}{MV_{t-1}}
$$

$$
-\frac{MV_{t-1}}{MV_t}(1+i_{t-1}^1)\frac{b_{t-1}^N(p_{t-1}^N\frac{p_t^{N-1}p_{t-1}^1}{p_{t-1}^N} + \sum_{j\in\{1,2,...N\}} p_{t-1}^j\frac{p_t^{j-1}p_{t-1}^1}{p_{t-1}^j}\kappa_{t-1}) + ... + b_{t-1}^1 p_{t-1}^1}{MV_{t-1}}
$$

Notice that the leading term in (18) represents the contribution of the new issuance (in period t) to the maturity structure in t. Short term debt contributes one period maturity whereas long term debt contributes ξ_t given by $\xi_t = \frac{(p_t^N N + \sum_{j\in\{1,2,...N\}} jp_t^j\kappa_t)}{(p_t^N + \sum_{j\in\{1,2,...N\}} p_t^j\kappa_t)}$. The term ω_t is the ratio of total issuance to the market value of debt. It captures the fact that higher issuance leads to a larger impact of the sharing rule in the maturity structure in t.

Note that the second line in (18) resembles the maturity structure in period $t-1$ (multiplied by $\frac{MV_{t-1}}{MV_t}(1+i_{t-1}^1)$). The only difference is that changes in bond prices (interest rates) between periods make the ratio $\frac{p_t^{j-1}p_{t-1}^1}{p_{t-1}^j}$ different from one. Therefore, changes in

[8] Note that the average maturity of government debt is essentially a weighted average of all the maturities of the payments promised by the government, the weights being the ratios of the market value in a particular maturity over the total market value of debt. For example, if we set $N = 2$ (to simplify) the maturity in period t is

$$
MAT_t = \frac{1}{MV_t}(b_t^N(p_t^N N + (1p_t^1 + 2p_t^2\kappa_t)) + b_{t-1}^N(p_t^1(1+\kappa_{t-1}) + 1b_t^1 p_t^1)
$$

This shows the way we derive the formula in (17).

[9] As discussed previously it is common for coupons to be such that the bond trades at par. Given bond prices in t, (p_t^j) the coupon in period t can be backed by the following equality $1 = \sum_{j\in\{1,2,...N\}} p_t^j\kappa_t))$.

the time path of interest rates and the yield curve affect the maturity structure in t. The same principle applies to the last line in (18). The numerator of the ratio is close to the market value of government debt in $t-1$.

In order to estimate the coefficients of our short term sharing rule (equation (18)) we utilize a linearized version of equation (18). Linearization enables us to drop the history of debt issuances since up to the first order, the second term of (18) will be replaced by the period $t-1$ average maturity. We therefore apply a first order Taylor expansion centered around a flat yield curve (so that a term of the form $\frac{p_t^{N-1}p_{t-1}^1}{p_{t-1}^N}$ is zero at the approximation point) and a constant growth of the quantity $\frac{MV_{t-1}}{MV_t}$ denoted by g. We get the following expression:

$$
\begin{aligned}
(19)\qquad MAT_t - MAT &= s^1(\omega_t-\omega) + (s_t^1 - s^1)\omega + (1-s^1)\omega(\xi_t-\xi) \\
&+(1-s^1)\xi(\omega_t-\omega) - (s_t^1-s^1)\omega\xi \\
&(MAT-1)g(i_{t-1}^1 - i^1) + (MAT-1)(1+i^1)(g_t-g) + (1+i^1)g(MAT_{t-1}-MAT) \\
&+g(1+i^1)\frac{b^N}{MV}(N-1)p^N(\frac{N(i_{t-1}^N - i)}{1+i} - \frac{(N-1)(i_t^{N-1}-i)}{1+i} - \frac{(i_{t-1}^1-i)}{1+i})+ \\
&+\sum_{j\in\{1,2,\dots N\}} g(1+i^1)\frac{b^N}{MV}(j-1)p^j\kappa(\frac{j(i_{t-1}^j-i)}{1+i} - \frac{(j-1)(i_t^{j-1}-i)}{1+i} - \frac{(i_{t-1}^1-i)}{1+i})+ \\
&+g^2(1+i^1)\frac{b^N}{MV}(N-2)p^{N-1}(\frac{(N-1)(i_{t-1}^{N-1}-i)}{1+i} - \frac{(N-2)(i_t^{N-2}-i)}{1+i} - \frac{(i_{t-1}^1-i)}{1+i})+ \\
&+\sum_{j\in\{2,3,\dots N-1\}} g^2(1+i^1)\frac{b^N}{MV}(j-1)p^j\kappa(\frac{j(i_{t-1}^j-i)}{1+i} - \frac{(j-1)(i_t^{j-1}-i)}{1+i} - \frac{(i_{t-1}^1-i)}{1+i})+ \\
&+g^{N-1}(1+i^1)\frac{b^N}{MV}p^1(1+\kappa)(\frac{2(i_{t-1}^2-i)}{1+i} - \frac{(i_t^1-i)}{1+i} - \frac{(i_{t-1}^1-i)}{1+i})
\end{aligned}
$$

We use the above expression under the sharing rule (16) to match the maturity structure. The steps of the procedure we apply are the following: First, we take from the data the time series on issuances (ω_t in our notation), the market value of debt (MV_t), and the time series on interest rates and the maturity of government debt. With these objects the above expression (19) is used to construct the share s_t^1 with which we can match exactly the average maturity structure as in the data. Therefore we basically back out s_t^1 from

(19). In a second step, with our time series for s_t^1 we run equation (16) using a simple OLS.

Note that this procedure is essentially forcing s_t^1 to give us a perfect fit in terms of the maturity structure. The share s_t^1 therefore is not the one we would obtain from the CRSP data, rather it corresponds to artificial data (or simulated data since it derives from simulating the model under (19)). It differs from its counterpart in the CRSP because, as discussed, assuming that the government issues debt in one year and ten year maturities is too simplistic to fully capture debt management in practice. Moreover, it is worth noting that fitting the simulated data s_t^1 on the independent variables in (16) (the lagged share and debt to GDP ratio) could also be done through a standard simulated method of moments procedure. One could choose the parameters of equation (16) minimizing the errors between the maturity structure and its data counterpart. Our two step approach is simpler and is aimed at circumventing possible convergence problems in the numerical implementation of the simulated method of moments algorithm.

We summarize our estimates here: We obtain a value for ω_1 equal to 0.238, a first order autocorrelation coefficient of roughly 0.765 and a coefficient which governs the response of the share to the lagged debt to GDP ratio (ω_2) equal to -0.177. [10]

To close this section we report the results we obtained from estimating equation (16) directly from the data with OLS. The point estimates of ω_1, ρ_s, ω_2 were: 0.205, 0.761 and -0.166 respectively (all statistically significant). These estimates are close to our estimates from the simulated data. The differences of course can be attributed to our assumption that the government issues debt only in one year and ten year bonds. [11]

[10]Because the estimation is based on artificial data we omit descriptive statistics. However, based on our sample, all estimated coefficients were found to be statistically significant. The error term produces an estimated standard deviation of 0.09 implying that a substantial part of the variation in the share s_t^1 is non-systematic.

[11]The fact that the two sets of estimates are close suggests that our assumptions are reasonable as an approximation for the US debt management policy, or to put it differently, that with a one year and a ten year bond (under no buyback and with coupons) we obtain a share s_t^1 from (19)) which does not differ considerably from its data counterpart (or to the least it doesn't differ in the portion which can be explained by the independent variables in (16)). This is not an obvious result. In a later section we will demonstrate that when we lift the assumption of no buyback the estimates of equation (16) will change considerably, because the s_t^1 variable backed out from that model will behave very differently than its data counterpart.

Table 1: *Calibration Table*

Symbol	Variable	Value
ρ_g	First order autocorrelation	0.9
ρ_z	First order autocorrelation	0.82
β	Discounting factor	0.95
ρ	First order autocorrelation	0.765
ω_1	Sharing rule	0.95
ω_2	Response of the share to the lagged debt to GDP ratio	-0.177
ρ_τ	First order autocorrelation	0.94
ϕ	Response of tax rate to the excess of the government debt over steady state	0.17

3.3 Implications for Fiscal Insurance

The above results have several implications for the fiscal insurance properties of government debt management in the US. First and foremost, we have shown that a substantial part of the government's portfolio is composed by one year maturity debt. As the analysis of section 2 made clear holding short term, rather than long term, debt implies that a smaller portion of government deficit can be financed through the (endogenous) changes in bond prices which take place after a fiscal shock. Rather, we anticipate that a large part of the fiscal burden falls to (distortionary) taxes. This as we said is consistent with the existing empirical literature on the US data (for example Hall and Sargent (2010)).

Second, we have established that as government debt increases, the largest portion of the new issuance is devoted to long maturity (ten year) debt. This finding suggests that the fiscal insurance benefit is larger at high debt levels enabling the government to finance its deficits, in this case more smoothly (i.e. without relying on increases in taxes as much as it does when debt is low). Such a policy response may be optimal if one considers that with high debt and taxes, generating revenues from further increases in the tax rate may be difficult if, say, the economy is close to being on the wrong side of the Laffer curve. In any case the results suggest that the potential benefits of debt management in terms of fiscal insurance are potentially less modest when the debt level is higher.

In the next section we embed these findings in a dynamic stochastic general equilibrium model. We seek to investigate whether the behavior of the debt aggregate is affected by shifts in the debt management strategy (i.e. through changes in the central features we have identified in this section). For this purpose a quantitative model is necessary. We consider a model broadly similar to the models of Angeletos (2002), Marcet and Scott (2009), Faraglia et al. (2014).

4 Model

This section presents our formal economic framework. As discussed, our model is broadly similar to Angeletos (2002), Buera and Nicolini (2006) and Faraglia et al (2014 (b)).

4.1 Economic Environment

The economy is populated by a single representative household whose preferences over consumption, c_t, and hours worked, h_t, are given by $E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(h_t))$, where u is strictly increasing and strictly concave function and $0 < \beta < 1$ is the discount factor.

The economy produces a single good that cannot be stored. The household is endowed with 1 unit of time which it allocates between leisure and labour. Technology for every period t is given by:

$$c_t + g_t = z_t h_t \tag{20}$$

where g_t represents government expenditure (assumed to be stochastic and exogenous) and z_t represents total factor productivity in the model and is also assumed to be stochastic. As is customary in the literature we assume that shocks to government spending and technology are the only sources of uncertainty in the model.

4.1.1 The Government

The government engages in the following activities to finance spending: First, it levies distortive taxes τ_t on labor income and second, it issues debt in bonds of two different maturities. We summarize the debt issuance of the government with a vector $b_t = \{b_t^1, b_t^N\}$ where N denotes the long bond.

Following our notation in section 2 we let p_t^i be the price of a bond of maturity $i \in \{1, N\}$ with $p_t^0 = 1$. The government budget constraint may be written as:

$$\sum_{i=\{1,N\}} b_t^i p_t^i = b_{t-1}^1 + \sum_{j=1}^{N} \kappa_{t-j} b_{t-j}^N + b_{t-N}^N + g_t - \tau_t z_t h_t \tag{21}$$

The left side of equation (21) is the value of the bond portfolio issued this period. Notice that in the case of the long term bond the price p_t^N determines, given the quantity b_t^N and the (implicit) sequence of coupons κ_t, the amount of revenues raised through the long term asset by the government. The first term on the right hand side represents the fraction of debt outstanding which matures in t. It consists of the promised coupon payments on the long bonds issued between periods $t-1$ and $t-N$ (e.g. $\sum_{j=1}^{N} \kappa_{t-j}$ and of the principal (one unit of income) multiplied by the quantity of debt issued in $t-N$ (b_{t-N}^N)). Obviously, the price on any asset which matures in date t (p_t^0) is equal to one.

4.1.2 Household Optimization

The household's budget constraint is given by:

$$\sum_{i=\{1,N\}} b_t^i p_t^i = b_{t-1}^1 + \sum_{j=1}^{N} \kappa_{t-j} b_{t-j}^N + b_{t-N}^N + (1-\tau_t) h_t z_t - c_t \tag{22}$$

The term $(1-\tau_t)h_t z_t$ represents the household's net income. Moreover, since practically any bond issued by the government is bought by the household we keep our notation of the term b_t^i which in (22) represent household savings in maturity i.

Note that combining equations (21) and (22) we can obtain (20). This is obviously so because debt issued by the government is the asset held by the household and there is no other financial asset (i.e. one which involves only the private sector) in the economy. Moreover, the total income produced in the economy $z_t h_t$ is divided between household consumption c_t and government spending g_t.

The household's objective is to maximize its utility subject to the budget constraint (22). Standard results imply that the optimization can be represented through the following Lagrangian function:

$$\mathcal{L} = E_0 \sum_t \beta^t (u(c_t) - v(h_t) - \lambda_t (\sum_{i=\{1,N\}} b_t^i p_t^i - b_{t-1}^1 - \sum_{j=1}^{N} \kappa_{t-j} b_{t-j}^N - b_{t-N}^N - (1-\tau_t) h_t z_t + c_t))$$

where λ_t is the (Lagrange) multiplier which measures the marginal utility of wealth.

The first order conditions for the optimum are given by the following equations:

$$u_c(t) = \lambda_t \tag{23}$$

$$v_h(t) = \lambda_t(1-\tau_t)z_t \tag{24}$$

$$\lambda_t p_t^1 = \beta E_t \lambda_{t+1} \tag{25}$$

$$\lambda_t p_t^N = \beta E_t \lambda_{t+1}\kappa_t + \beta^2 E_t \lambda_{t+2}\kappa_t + ... + \beta^N E_t \lambda_{t+N}(1+\kappa_t) \tag{26}$$

where $u_c(t)$ represents the marginal utility of consumption in t and $v_h(t)$ is the analogous marginal disutility of work effort (hours). Equations (23) to (26) represent the optimality conditions with respect to c_t, h_t, b_t^1 and b_t^N. Notice that κ_t is not a choice variable for the household. As discussed previously the coupon rate is chosen by the government.

Equation (23) sets the marginal utility of consumption equal to the multiplier λ_t. (24) equates $v_h(t)$ to the net benefit of working given by the net income term $(1-\tau_t)z_t$ times the marginal utility of consumption. Rearranging these two equations we obtain:

$$\frac{v_h(t)}{u_c(t)} = (1-\tau_t)z_t \tag{27}$$

which is the familiar optimality condition giving that the marginal rate of substitution between consumption and hours is equal (at the optimum) to the net wage.

Moreover, notice that substituting (23) into (25) and making use of the fact that $u_c(t+1) = \lambda_{t+1}$ in period $t+1$, we get:

$$p_t^1 u_c(t) = \beta E_t u_c(t+1)$$

and dividing by the marginal utility we get:

$$p_t^1 = \beta E_t \frac{u_c(t+1)}{u_c(t)} \tag{28}$$

Note that (28) gives us the price of one year government debt, that we utilized in section 2. In the context of the household's optimal program we derived here, it suggest that the price (the inverse of the gross rate of the return), is equal to the marginal rate of substitution of consumption between t and $t+1$ where the weight attached to $t+1$ consumption is basically the factor β. According to this equation the household, which sacrifices p_t^1 units of c_t for one unit of consumption tomorrow, optimizes if the condition in (28) holds.

To derive the long bond price we now combine (26) with (23). Following the same procedure of substituting in (26) the marginal utility we get:

$$p_t^N = \beta^N E_t \frac{u_c(t+N)}{u_c(t)} + \sum_{j=1}^{N} \kappa_t \beta^j E_t \frac{u_c(t+j)}{u_c(t)} \tag{29}$$

which suggests that the price of a non zero coupon bond today is equated to the future flows of income it promises, appropriately discounted through the factors $\frac{u_c(t+j)}{u_c(t)}$, $j = 1, 2, ...N$.

Similar arguments to the one we invoked in this section may be applied to price all of the assets we considered in section 2 (e.g. long term bonds with and without buyback).

4.1.3 Tax Policies

We had previously explained that in the context of economic models which study debt management, it has been customary in the literature (see Faraglia et al. (2014(b))) to assume that the government sets optimally the tax schedule and chooses the portfolio consisting of short and long term debt. Here rather than assuming a 'benevolent planner' as Faraglia et al. (2014 (b)) do, we summarize the institutions into a simple tax rule that can be mapped into the US data. We postulate that [12]:

$$\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau)\overline{\tau} + (1-\rho_\tau)\ \phi(\frac{MV_{t-1}}{GDP_{t-1}} - \overline{\frac{MV}{GDP}}) + \epsilon_\tau \tag{30}$$

therefore we assume that the tax rate is a function of its lagged value and responds to the excess of the market value of government debt over a predetermined steady state level

[12] The term ϵ_τ is a tax shock which we will not be present in the model. Here we include this term to make clear that we do not claim that a tax rule of the form (30) would fit perfectly the empirical observations

($\overline{\frac{MV}{GDP}}$) with a coefficient ϕ. The value of ρ_τ gives the persistence of the tax rate, e.g. the time horizon over which an increase in the market value of government debt (above normal) will provoke a rise in the tax rate, to satisfy intertemporal solvency. In the case where $\rho_\tau < 1$ the tax rate displays mean reversion suggesting that after a certain time period taxes are expected to return to their steady state value of $\overline{\tau}$ if government debt to GDP is at the level of $\overline{\frac{MV}{GDP}}$, that is if the fiscal adjustment is sufficient to bring the debt stock to its 'normal' level.

Note that we do not take equation (30) directly to the US data. Rather we rely on existing estimates from the literature to pin down the values of the parameters ρ_τ and ϕ. In particular the estimates we utilize are taken from Leeper et al. (2013) and suggest that setting $\rho_\tau = 0.94$ and $\phi = 0.17$ is a good approximation of the US fiscal policy rule. Note that according to these estimates the value of ϕ implies that a rise of the market value of debt over GDP by one percentage point leads to an increase in the tax rate by 0.01 percentage points. This increase persists over several periods since also ρ_τ is of a high value (close to one).

It is worth noting at this point that in models of optimal policy as in Faraglia et al. (2014 (b)) tax rates typically follow a stochastic process close to a random walk. This is to say that when taxes are set optimally and do not necessarily conform with rule (30), they nevertheless display substantial persistence (or to put it differently a coefficient ρ_τ which is close to one). Therefore it seems that fiscal policy in the US conforms with this principle.

Finally notice that given the above condition, and under the assumption that in our economy debt management will impact the behavior of the market value of debt, we anticipate that different sharing rules of the form (16) will exert a different influence on the tax rate and therefore on the behavior of the private sector and on the economic aggregates such as hours, consumption and interest rates. The purpose of our exercise in the following section is to trace this impact.

4.2 Solution Details

4.2.1 Solution Method

We solve the model by applying the parameterized expectations algorithm (hereafter PEA) of den Haan and Marcet (1994) (also described in Judd et al. (2010)). This procedure is to solve the model based on the system of optimality conditions (equations (23) to (26)) and to approximate any term which involves a conditional expectation, by polynomials formed with the state variables of the model. More specifically the system of equations which has to be solved consists of the economy's resource constraint (20), the government budget constraint (21), the optimality condition which determines hours worked $\frac{v_h(t)}{u_c(t)} = (1-\tau_t)z_t$, the tax rule (30) and the expressions for short term and long term bond prices derived above. Moreover, the policy rules for taxes and debt management must be accounted for. For expositional purposes we repeat here the system which we want to approximate numerically.

$$\frac{v_h(t)}{u_c(t)} = (1-\tau_t)z_t \tag{31}$$

$$c_t + g_t = z_t h_t \equiv GDP_t \tag{32}$$

$$p_t^1 = \beta E_t \frac{u_c(t+1)}{u_c(t)} \tag{33}$$

$$p_t^N = \beta^N E_t \frac{u_c(t+N)}{u_c(t)} + \sum_{j=1}^{N} \kappa_t \beta^j E_t \frac{u_c(t+j)}{u_c(t)} \tag{34}$$

$$\sum_{i=\{1,N\}} b_t^i p_t^i = b_{t-1}^1 + \sum_{j=1}^{N} \kappa_{t-j} b_{t-j}^N + b_{t-N}^N + g_t - \tau_t z_t h_t \tag{35}$$

$$\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau)\overline{\tau} + (1-\rho_\tau)\,\phi\left(\frac{MV_{t-1}}{GDP_{t-1}} - \overline{\frac{MV}{GDP}}\right) + \epsilon_\tau \tag{36}$$

$$MV_t = b_t^1 p_t^1 + \sum_{j=0}^{N-1}\sum_{k=1}^{N-j} \tilde{p}_t^k \kappa_{t-j} b_{t-j}^N + \sum_{j=0}^{N-1} \tilde{p}_t^j b_{t-j}^N \tag{37}$$

$$\tilde{p}_t^k = \beta^k E_t \frac{u_c(t+k)}{u_c(t)} \tag{38}$$

$$s_t^1 = \omega_1 + \rho s_{t-1}^1 + \omega_2 \frac{\text{Debt}_{t-1}}{\text{GDP}_{t-1}} \tag{39}$$

There are several noteworthy features: First, note that in the above system of equations we have included the definition of the market value of government debt which is the appropriately discounted present value of all debt outstanding in period t (after the issuance and the redemption of maturing debt in that period). Second, notice that in order to determine the market value we utilize bond prices $\tilde{p}_t^k$ (k being the maturity of a claim) as opposed to using the prices p_t^1 and p_t^N derived previously. The reason is that it is simpler (in terms of the numerical algorithm we use to solve the model) to strip the coupons of each bond and to price coupon and principal separately. Therefore $\tilde{p}_t^k$ is basically the price of a claim in t, which delivers one unit of consumption in period $t+k$. Applying our previous arguments it is straightforward to show that this price is equal to $\beta^k E_t \frac{u_c(t+k)}{u_c(t)}$.

Given the above expressions we solve the model applying the PEA. This numerical procedure consists of approximating all of the terms which involve a conditional expectation in t (effectively bond prices) with polynomials composed by the state variables of the model. Let X_t be a vector which contains all the relevant state variables. [13] In essence it is sufficient to approximate the following terms:

$$E_t u_c(t+i), \quad 1, 2, ...N$$

as functions of X_t.

Let $\Phi(X_t, \delta^i)$ denote the approximation of $E_t u_c(t+i)$ where Φ is the polynomial function, and δ^i is a vector of coefficients on the state variables applying to maturity i. Note that the index i is meant to capture that the true coefficients δ differ between maturities. Our numerical procedure is basically to start with an initial guess on the vectors δ^i to solve the model for a large number of periods S, and use the simulations of the terms $u_c(t+i)$ to project them on the state variables X_t and update the value of the coefficients. The procedure is described thoroughly in Judd et al. (2010). For the sake of the exposition we provide here an algorithm to solve the model.

Step 1 Choose a simulation length S and draw a sequence of government spending and

[13] Note that here state variables are all predetermined variables (for example the lagged tax rate and the lagged debt to GDP ratio but also all lags of bond quantities issued), the current realizations of the level of technology z_t and the value of government spending g_t.

technology shocks. Choose a specification (order and family) for the polynomial Φ and set the initial coefficients δ_0^i for $i - 1, 2, ...N$. Also pick an initial value for the state vector (X_1).

Step 2 Given these objects solve the system of equations (31) to (39) at each date $t = 1, 2, ...S$ given the realization of the state vector X_t. Use the approximations $\Phi^1(X_t, \delta_0^i)$ to compute a time path for consumption, bond holdings and the Lagrange multiplier.

Step 3 Use the simulated path to update the coefficients and δ_0^i. First, use the paths of consumption to construct the expressions $u_c(t+i)$. Then, regress these expressions on the polynomials of the state variables to update the coefficients. For example, we run a regression of $u_c(t+1)$ on the states to get a new value $\hat{\delta}^1$ (and analogously for every conditional expectation). This regression is effectively isolating the components of $u_c(t+1)$ which are contained in the date t information set. In other words, our approximation is essentially of the conditional expectation of these terms as a function of the state variables.

Step 4 Compute the vector of coefficients to use in the next iteration as:

$$\delta_1^i = \delta_0(1-\mu) + \mu\hat{\delta}^i$$

where $\mu \in (0,1)$. Iterate on Steps 1 to 4 until convergence is achieved (until the coefficients δ_k^i and δ_{k-1}^i are close to each other).

Our convergence criterion is such that the maximum (over all i) percentage difference in the coefficients in two successive iterations is less than 0.0001. This choice follows Faraglia et al. (2014 (b)).

4.2.2 Calibration

In order to solve the model, we must first specify the exact form of the household's utility function and also set the values for every structural (deep) parameter. In this subsection we briefly mention our targets and choices for these values and functional forms.

First, we set β equal to 0.95. This gives an average value for the short term interest rate equal to 5.26% which closely corresponds to the average of the annual interest rate in our sample period in the data. Second, we assume that the household's utility is given by:

$$\log\,(c_t) - \chi \frac{h_t^{1+\gamma}}{1+\gamma} \tag{40}$$

We fix the value of γ to one (implying a unitary elasticity of labor supply) and we pick a value of χ so that the model produces steady state hours worked of one third. Note that assuming that the utility of consumption is represented by the log function is standard in the literature. Moreover, we take the exact specification of utility and the value of γ from Smidt-Grohe and Uribe (2004).

We further assume (following Smidt-Grohe and Uribe (2004)) that the ratio of government spending to output in the steady state is equal to 20%. It is also assumed that the government debt to GDP ratio is 60%. [14] Given this value we find from (16) the share of short term bonds issued in each period. We thus compute the quantities of one year and ten year bonds issued in each period in the steady state.

The stochastic processes for government spending and technology are given by the following equations:

$$g_t = g_{t-1}^{\rho_g} \overline{g}^{1-\rho_g} e^{\epsilon_{g,t}} \tag{41}$$

$$z_t = z_{t-1}^{\rho_z} \overline{z}^{1-\rho_z} e^{\epsilon_{z,t}} \tag{42}$$

where $\overline{g}$ and $\overline{z}$ represent the steady state levels of spending and technology respectively (the latter is normalized to unity). Following Smidt-Grohe and Uribe (2004) we set the variances of the innovations to government expenditures and technology to be equal to 0.03 and 0.02 respectively. Further on, we set the first order autocorrelation coefficients

[14] Since we do not rely on a linear approximation around the steady state and rather solve the model with global methods, we can capture the importance of debt management on the behavior of the economy very accurately at any point in the state space which is visited by the simulations.

ρ_g and ρ_z equal to 0.9 and 0.82 respectively.

To calibrate the coupons we make the following assumptions: First, we assume that long term bonds pay a constant coupon κ in every period. Second, we set the value from κ so that in the steady state the price of the ten year bond is equal to one (i.e. the bond trades at par). We previously argued that the US government issues coupons on long term bonds to ensure that bond prices are aligned with the principal paid at maturity. However, it is important to note that actual bonds in the CRSP data do not trade (most of the time) exactly at par, rather they trade close to par. Since it is well known that the model which we utilize will not produce large swings in asset prices (large changes in the slope of the yield curve) we can claim that the constant coupon assumption is a good approximation of reality. [15] We make this assumption here to simplify the computations; we basically do not have to solve each period for the coupon value which gives a price exactly equal to one. However, note that our derivations in the previous section continue to hold, the only difference being that in the system of equations, coupons are no longer indexed by the time period of the bond issuance.

4.2.3 Debt Limits

Given the tax policy rule (30) and the assumed parameters, we can verify that in equilibrium government debt is not an explosive process. However, since the changes in the debt management practice we study below may have substantial effects on the dynamics of the debt to GDP ratio we cannot rule out that for some of the policies considered, debt will become explosive. This is more of an issue in the application of the numerical algorithm we described previously. Indeed, given a set of initial conditions for the coefficients δ^i it could be that the debt to GDP ratio increases considerably in some parts of the simulation or becomes very negative in other parts (even if the true model equilibrium features a stationary ratio). In such cases we would not be able to approximate the equilibrium well.

To be able to contain the numerical solution we add two exogenous (ad hoc) limits

[15] This is a claim which we have verified with the simulations. Indeed we obtain in equilibrium bond prices which are close to par no matter the state of the economy.

on the market value of government debt. We first assume that there is an upper bound represented by $\overline{M}$ such that $MV_t \leq \overline{M}$ for all t and also a lower bound $\underline{M}$ such that $MV_t \geq \underline{M}$. These bounds are common in the literature of optimal policy models and therefore, following Faraglia et al. (2013), we set $\overline{M}$ be equal to 100% of steady state GDP (so that the debt to GDP ratio can be at most 100%) and $\underline{M} = 0$ so that the government cannot take a negative position in the bond market (i.e. lend to the private sector).

Practically, in terms of our numerical solution, the inclusion of the bounds means the following: In cases where the market value to GDP increases above the upper bound in a given period t, the tax rule (30) does not apply. When this happens in our simulations we need to find a tax rate which keeps the market value equal to $\overline{M}$. The same holds for cases where the market value drops below zero. However, we note that this problem applies usually out of equilibrium. In the models we analyze below the bounds bind extremely rarely in our simulations.

Finally, we note that though these bounds serve to help us deal with a numerical difficulty, in fact they are realistic to assume. First because we do not observe (in the historical data) the US government holding savings (and therefore it makes sense to impose a lower bound of zero), and second, because it is well known that a key institutional feature of debt issuance in the US is the presence of a legislated upper bound on the value of government liabilities (so that also the inclusion of the upper bound on the market value is sensible). Hence the debt limits could be viewed also as an institutional feature of debt management.

5 Results

This section contains our main results. After studying the properties of the benchmark model, we evaluate whether changes in the debt management rule may have significant effects on the behavior of the economy. We summarize in this section these effects through graphs showing sample paths from simulations of the model, and through measuring the sample moments of key statistics (such as output consumption, taxes and the market value of debt).

The changes in the debt management rule which we evaluate are the following: First we eliminate the dependence of the issuance on the debt to GDP ratio. We therefore let the sharing rule (in the absence of shocks to the share) be constant over time. As we will show when we eliminate the debt dependence our parameters lead to a share of short term debt which is equal to one. Therefore, this model is essentially a model where all government debt is short term.

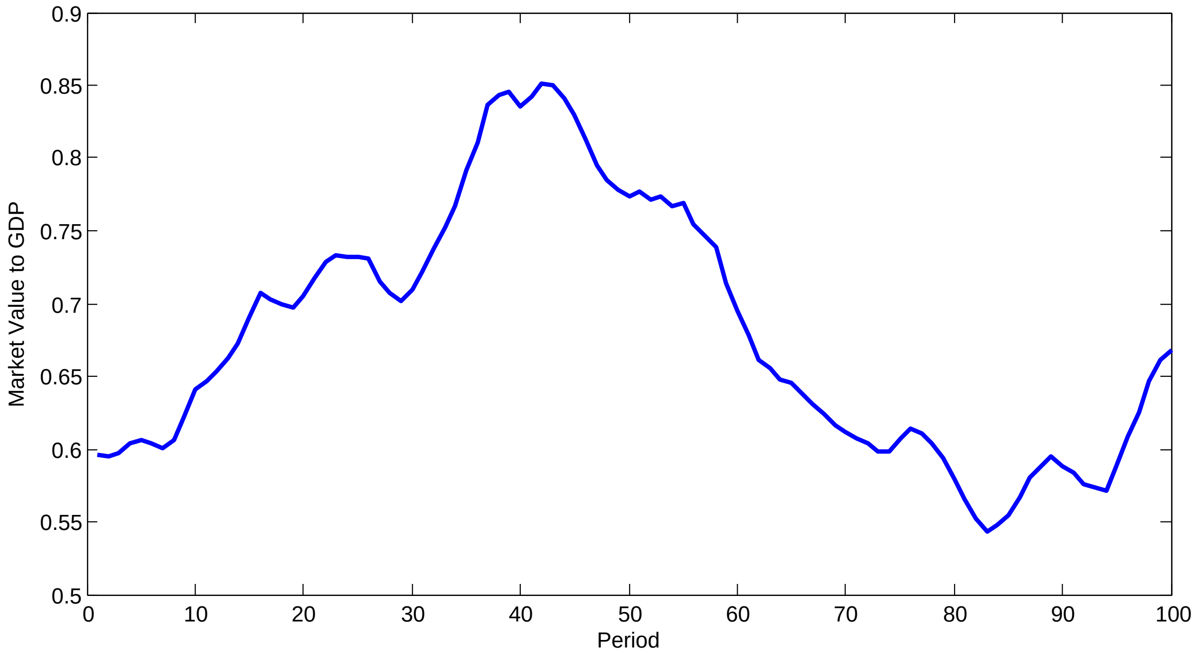
Second, we consider a reform of the debt management practice whereby the government, rather than redeeming its debt at maturity, buys back its outstanding obligations in every period.

As discussed in the introduction, the first of these experiments follows the structure shared by many macroeconomic models where it is assumed that government debt is of one period (either year or quarter). The second is a common assumption made in models of optimal fiscal policy (for example Angeletos (2002)). We therefore try to utilize our framework to study the economic significance of these alternative setups and compare them to the current debt management strategy followed by US authorities.

5.1 Baseline Debt Management

We illustrate in this paragraph our results from the baseline calibration of the model. In Figure 6 we plot a sample (100 model periods) from the market value of government debt denominated by GDP. As is illustrated in the figure the market value initially at 60% (our starting value) rises to roughly 85% after a few model periods, and subsequently decreases to about its steady state value.

Figure 6: **Market Value of Debt / GDP (Baseline Model)**



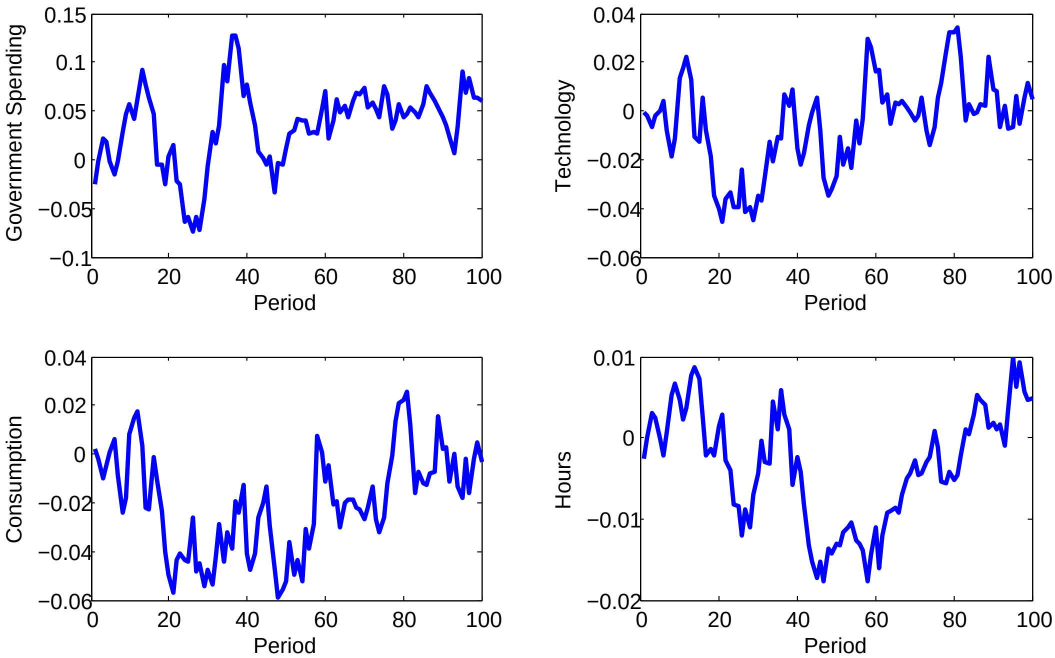
Notes: The figure shows a simulated path of the market value of debt to GDP ratio from the baseline model.

Figure 7 shows the values of government spending, productivity (top left and right panels) and also the values of consumption and hours over time (bottom left and right). All quantities are expressed in percentage deviation from their steady state levels. As is clearly illustrated higher government debt is due to a run of high spending shocks initially. However the impact of spending on debt is more persistent and even when expenditure levels are more moderate government debt continues to rise.

To understand this behavior, note that increases or decreases of debt are explained jointly by expenditures and tax revenues. Since tax rates are slow to adjust (we have assumed a very persistent component on taxes) the market value may continue to increase until tax revenues are substantially higher. On the other hand, towards the end of the sample, taxes remain high long enough for the market value to decrease even though government spending is above its mean value.

The bottom panels of Figure 7, which show private consumption and hours, suggest

Figure 7: **Simulated Paths of Economic Variables**

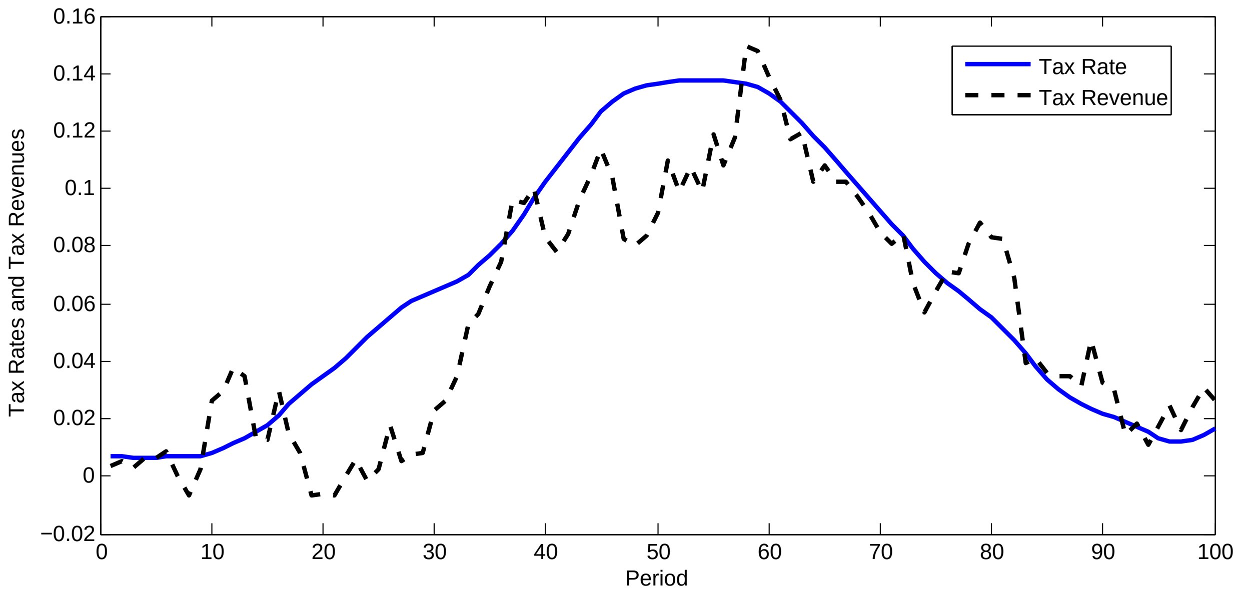


Notes: The figure shows a simulated path from the baseline model. The top-left panel represents government spending (in deviation from the steady-state). The top-right panel shows labor productivity. Bottom left and right panels plot the behavior of private sector consumption and hours respectively.

that consumption drops when spending is high (this can be attributed to the resource constraint) and rises when productivity improves. Notice that, even if at high frequencies the behavior of consumption is chiefly affected by fluctuations in technology, a significant portion of its long term variability is determined by the debt level. Therefore, since debt and spending are high over the sample period, private sector consumption is below the steady state value. Similar effects explain the behavior of hours worked. As is evident from the figure, hours are highly correlated with technology. However, for most of the sample (particularly around period 50) hours are considerably lower than the mean value.

To further explain these properties in Figure 8 we show the behavior of the tax schedule (solid line) and the analogous behavior of the tax revenues over the sample period. Notice that taxes increase towards the middle of the sample (reflecting the course of the market value of debt) and subsequently drop towards the end of the sample. The tax rate (given its specification) exhibits inertia in response to the market value. Notice also that, even though the total revenues of the government are affected by hours and productivity, the trend of this variable is primarily determined by the behavior of the tax rate.[16]

Figure 8: **Tax Rate and Tax Revenue (Baseline Model)**



Notes: The figure shows the behavior of the tax rate and the analogous behavior of the total revenues collected by the government over a period in the baseline model. Tax rates are represented by the solid line (blue), tax revenues by the dashed line (green).

These results confirm our previous remarks, that the behavior of hours worked is primarily affected over the medium to long run by variations in the tax rate.

5.1.1 Moments

In Table 2 we show sample standard deviations and first order autocorrelations for consumption, hours, the market value of debt, the share of short term debt and the tax rate. Standard deviations are expressed relative to GDP.[17]

[16]We define tax revenues as $\tau_t h_t z_t$.

[17]The moments reported here are based on our simulations of the model. We use 100 samples of 1000 periods each to solve the model with PEA. Therefore the standard deviations and the first order autocorrelations are essentially averages of these statistics over 100 samples.

Table 2: *Moments: Long samples*

	σ_H	σ_C	σ_{MV}	σ_{share}	σ_{Tax}	$\sigma_{Revenue}$
1	0.5426	1.0143	8.0115	18.7178	2.7933	0.8553
2	0.5722	1.0135	8.2086	0	3.0120	0.8886
3	0.4641	1.0167	6.1736	3.7410	2.4498	0.7223
	ϕ_H	ϕ_c	ϕ_{MV}	ϕ_{share}	ϕ_{Tax}	$\phi_{Revenue}$
4	0.8296	0.9432	0.8318	0.9936	0.9955	0.9965
5	0.8380	0.9472	0.8386	NA	0.9998	0.9956
6	0.8199	0.9416	0.8229	0.9934	0.9966	0.9937

Notes. Standard deviations (σ) and first order autocorrelations (ϕ) for consumption (C), hours (H), the market value of debt (MV), the share of short term debt (share), the tax rate (Tax) and the tax revenues (Revenue). Standard deviations are expressed relative to GDP.

There are several noteworthy features: First note that since the model's horizon is annual, the empirical counterpart for these statistics is not the usual facts concerning the US business cycle. Moreover, since our focus here is not to match any data moments, we have omitted these objects from the table. However, it seems obvious given the results presented that the joint impact of shocks to technology and government expenditures cannot generate substantial variation to GDP which is not surprising given that the model does not include capital and therefore investment. For the same reason consumption is as volatile as GDP in the model.

Second, note that the model variables can be divided (along the lines suggested by the table) into two groups: Variables which exhibit moderate persistence and low volatility, and variables which show very high persistence (a first order autocorrelation near one) and high volatility relative to GDP. To the first group belong hours and consumption, and to the second group belong the market value of debt, the tax rate and the share of one year bonds over the total issuance. Clearly the direct association between the share, the tax rate and the market value is the driving force behind this implication of the model. Since debt is very persistent, so is the tax rate, and the share given our assumed specifications for these objects.

This model implication is important for the following reason: It is well known (see Marcet and Scott (2009)) that in models of incomplete financial markets, government debt and tax rates display substantial persistence. The intuition is that if there is ever a shock to the government budget which causes an increase in deficit, the debt level will rise near permanently unless taxes are frontloaded and rise considerably to deal with the shock. In contrast, if markets are complete, which coincides with saying that a large portion of the government deficit can be financed through bond returns, the persistence of the market value is less. In the latter case we would anticipate the persistence of the tax rate and that of the debt level to be roughly equal to the persistence of the stochastic processes of spending and technology shocks. Our findings here can be interpreted as an indication that in our model financial markets are incomplete. [18] We will return to this feature in a subsequent section.

5.2 No Debt Dependence

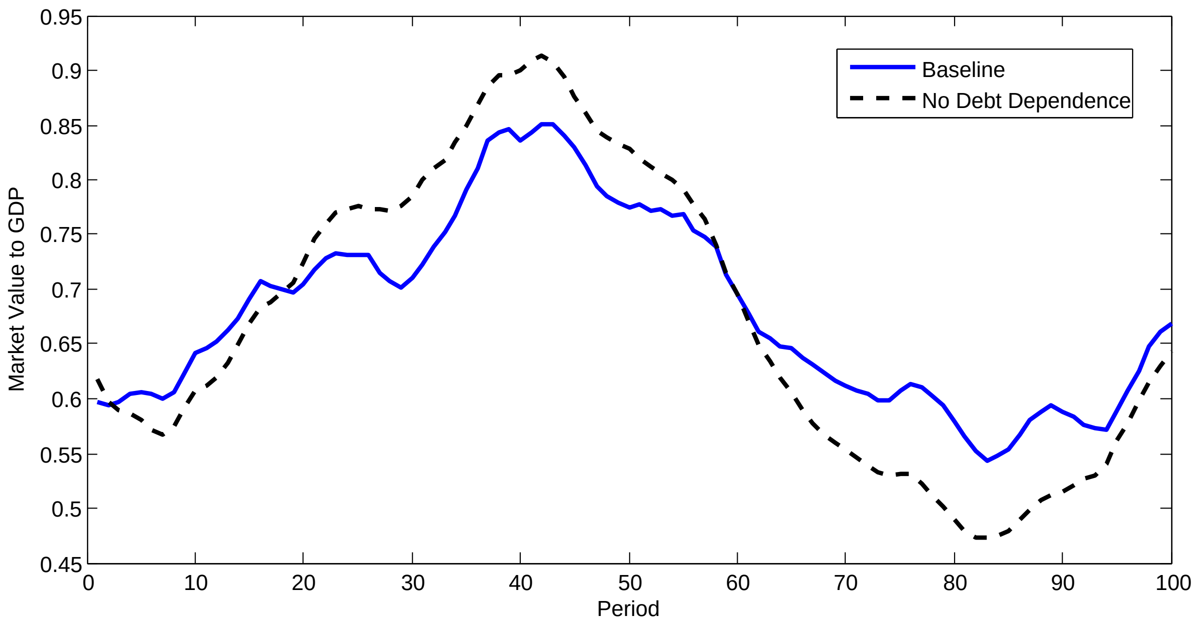
We now turn to consider the implications of altering the debt management practice. Our first experiment is to eliminate the dependence of the sharing rule on the debt to GDP ratio. Notice that since we have assumed no shocks to the share, and therefore the market value of debt is the only factor which causes the share to fluctuate over time, ruling out the influence of debt means that the share is effectively constant over time. Moreover, notice that given the specification in (16) and if we assume that $\omega_2 = 0$, we get in the steady state: $s_1 = \frac{\omega_1}{1-\rho_s}$. Our estimated values for ω_1 and ρ_s (0.2380 and 0.765) respectively give us a share which is slightly greater than one. In order to simplify we let the share be constant and equal to one at all times in our simulations, meaning that the model of this section is really a model where all government debt is one year.

In Figures 9, 10 and 11 we illustrate the behavior of the market value to GDP ratio

[18] This is in line with our earlier finding that the debt management practice in the US and hence the one we assume in the model, does not yield a substantial gain in terms of fiscal insurance. In essence, though we do not assume that government debt is state contingent (as would be the case if we had assumed the presence of Arrow-Debreu securities) the government can in principle replicate complete markets through managing the maturity of debt. This is the theoretical result in Angeletos (2002) who shows that a portfolio of only long term debt (and several times as large as GDP) can complete the market. Obviously our estimates imply a more balanced portfolio (i.e. one which includes short term debt). So markets are incomplete in the model.

(figure 9), the tax rate (figure 10) and the tax revenue (figure 11). The solid blue lines represent these objects in the simulations of the baseline version of the model (see previous paragraph). The dashed lines represent the analogous objects for the model of this section. As is shown in the figures, making the share constant and equal to one increases the volatility of the market value and the tax rate. In each of the graphs shown the dashed lines exhibit a steeper rise at the beginning of the sample which leads to a higher value around period 50 (the maximum debt to GDP is 90% whereas it is 85% under the baseline model). A similar result is obtained for the tax rate and the tax revenues of the government.

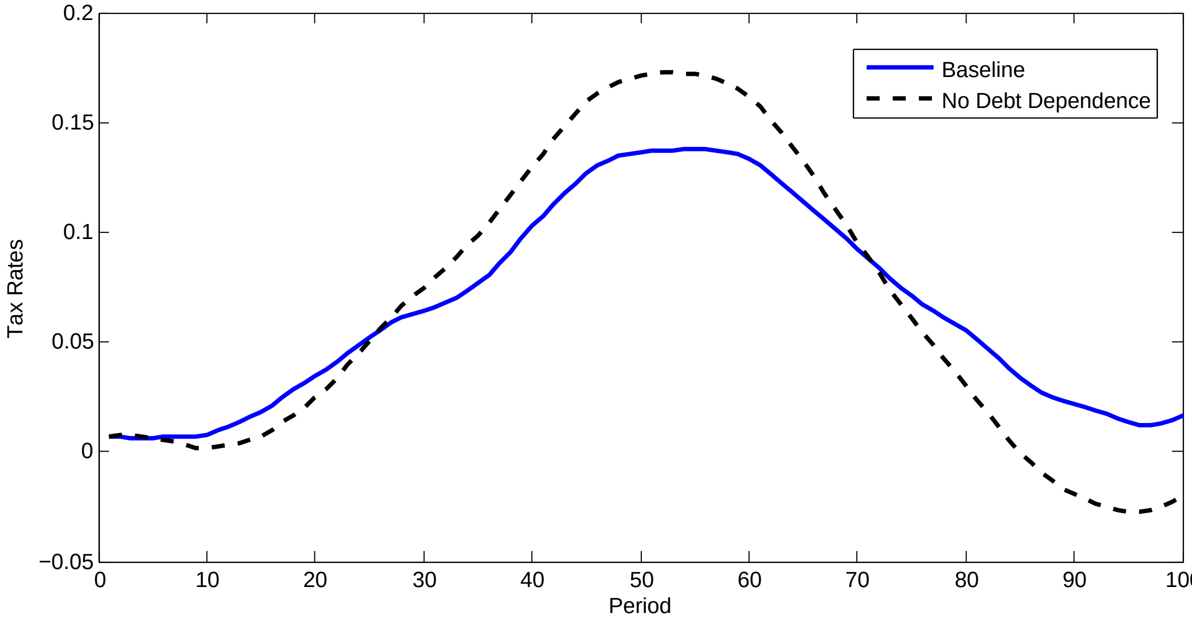
Figure 9: **Market Value of Debt / GDP**



Notes: The figure shows the behavior of the market value of debt to GDP ratio. The solid line represents the baseline model (blue). The dashed line represents the 'zero dependence' model (green).

This model implication can be better understood if viewed through the lens of the fiscal insurance principle discussed in section 2. As we argued in that section, when government debt is long term, part of the rise in the value of the deficit can be financed through the ensuing drop in asset prices. When debt is high and rising (due to spending

Figure 10: **Tax Rate**

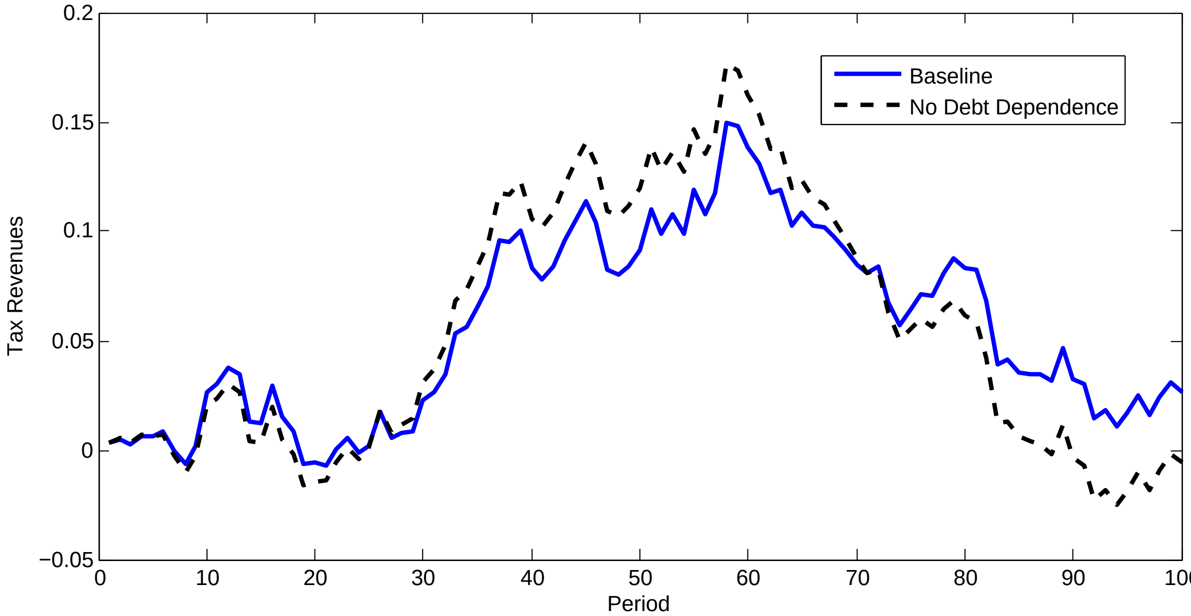


Notes: The figure shows the behavior of the tax rate. The solid line represents the baseline model (blue). The dashed line represents the 'zero dependence' model (green).

shocks) the dynamics of debt will be more moderate the longer the maturity structure is, and more explosive the shorter the average maturity is. Here we have assumed that the share of short term debt over total issuance is equal to one which leads to a maturity equal to one in all periods. Under the benchmark model the maturity of debt is greater than one; on average in the simulations it is 6.5 years since by construction it matches the average maturity of the US data. Therefore, under the benchmark model, shocks to government spending and productivity shocks, are partly absorbed through changes in the market value of government debt which mitigate the required increase in taxes.

To further clarify this reasoning we find it purposeful to repeat here the argument of fiscal insurance applied to the stochastic model of this section. Making use of equation (15) we can derive the intertemporal budget constraint of the government, which holds

Figure 11: **Tax Revenue**



Notes: The figure shows the behavior of the tax revenue. The solid line represents the baseline model (blue). The dashed line represents the 'zero dependence' model (green).

as an equality, as follows:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c(t+j)}{u_c(t)} (-g_{t+j} + \tau_{t+j} z_{t+j} h_{t+j}) = \tag{43}$$

$$b^1_{t-1} + \kappa \sum_{j=1}^{N} \beta^{N-j} E_t \frac{u_c(t+N-j)}{u_c(t)} b^N_{t-1} + \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} b^N_{t-1}$$

$$\kappa \sum_{j=1}^{N-1} \beta^{N-1-j} E_t \frac{u_c(t+N-1-j)}{u_c(t)} b^N_{t-2} + \beta^{N-2} E_t \frac{u_c(t+N-2)}{u_c(t)} b^N_{t-2} + ... + b^N_{t-N}(1+\kappa)$$

Again the terms on the right side of the above equality are key to understanding the patterns we showed in Figures 9, 10 and 11. Under the benchmark model, because the share of short maturity (one year) debt is persistently below one, the government portfolio consists of both short term and long term debt. Therefore, in the case of a rise in spending, drops in the expected growth rates of marginal utility will absorb part of the shock. However, when the share of short term debt is always equal to one (as is the case in the model of this section then the sole term which according to the right hand side of (43) influences the intertemporal constraint is b^1_{t-1}. This term offers no insulation to the government's budget since it is not multiplied by an endogenous asset price. Consequently, a larger increase in tax revenues is required for the intertemporal constraint to hold as an equality which explains the pattern of the tax rate and the tax revenues documented in the Figures 10 and 11. [19]

To show that these channels are present in our simulations we plot in Figures 12 and 13 the impulse responses of $E_t \frac{u_c(t+i)}{u_c(t)}$ for $i = 1$ (solid lines) $i = 5$ (dashed lines) and $i = 9$ (crossed lines) to a one standard deviation increase in spending (Figures 12) and a one standard deviation drop to technology (Figures 13). The responses graphed are in percentage deviation from the steady state. Moreover, the starting value for the debt to GDP ratio in all cases is the steady state value. As the figures show shocks which increase the deficit lead to a drop in expected marginal utility across all maturities, implying that the resulting movements in bond prices are lined up with the fiscal insurance argument explained above. [20]

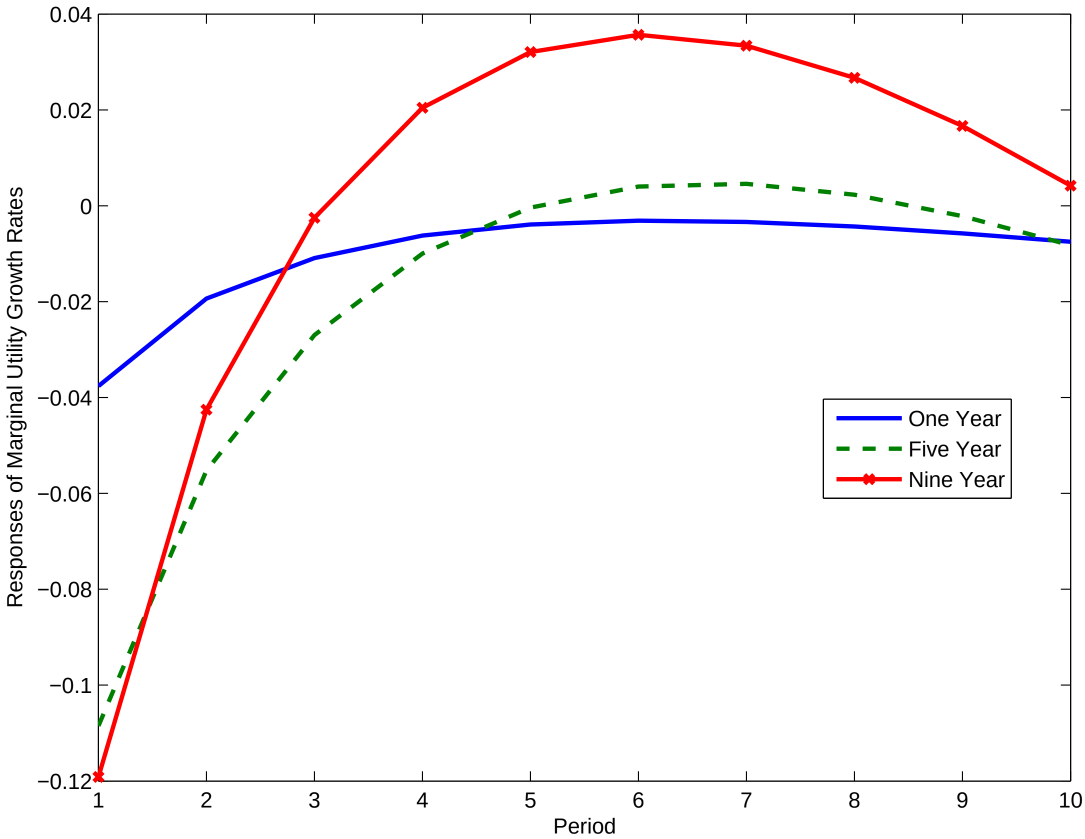
5.2.1 Moments

Rows 2 and 4 of Table 2 show the set of moments for the model of this paragraph. Notice that when summarized in these moments, it is evident that for most economic variables the different debt management rule we assume here does not have a considerable impact.

[19] A similar argument but with a very complex derivation can also be applied to the market value (see for example Hall and Sargent (2010)). In particular, it is possible to show that the market value in t is proportional to the right hand side of (43) since these terms govern the government's current and future debt refinancing needs. Rather than deriving this explicitly it is perhaps more straightforward to argue that since taxes rates are linked to the market value in the economy, the more modest increase in the tax rate under the benchmark model, should be echoed to a more modest increase in the market value of government debt. This also explains the pattern in Figure 9.

[20] Note that we are interested in the effect of shocks on bond prices in the first period. In all cases the ratio $E_t \frac{u_c(t+i)}{u_c(t)}$ drops on impact. The fact that in some cases the marginal utility growth rate may cross the zero line a few periods after the shock should not be surprising. In equilibrium asset prices are not only a function of technology and spending (whose movements are given exogenously by their first order autoregressions), but they are also a function of the responses of taxes and the distribution of past issued debt. These objects which evolve endogenously in the model typically have non-monotonic adjustment paths to disturbances leading to complex dynamics in the expected consumption paths. Though we believe that these effects are interesting in their own right it is beyond the scope of this thesis to analyze them.

Figure 12: **Response of Expected Marginal Utility Growth Rates to the Spending Shock**

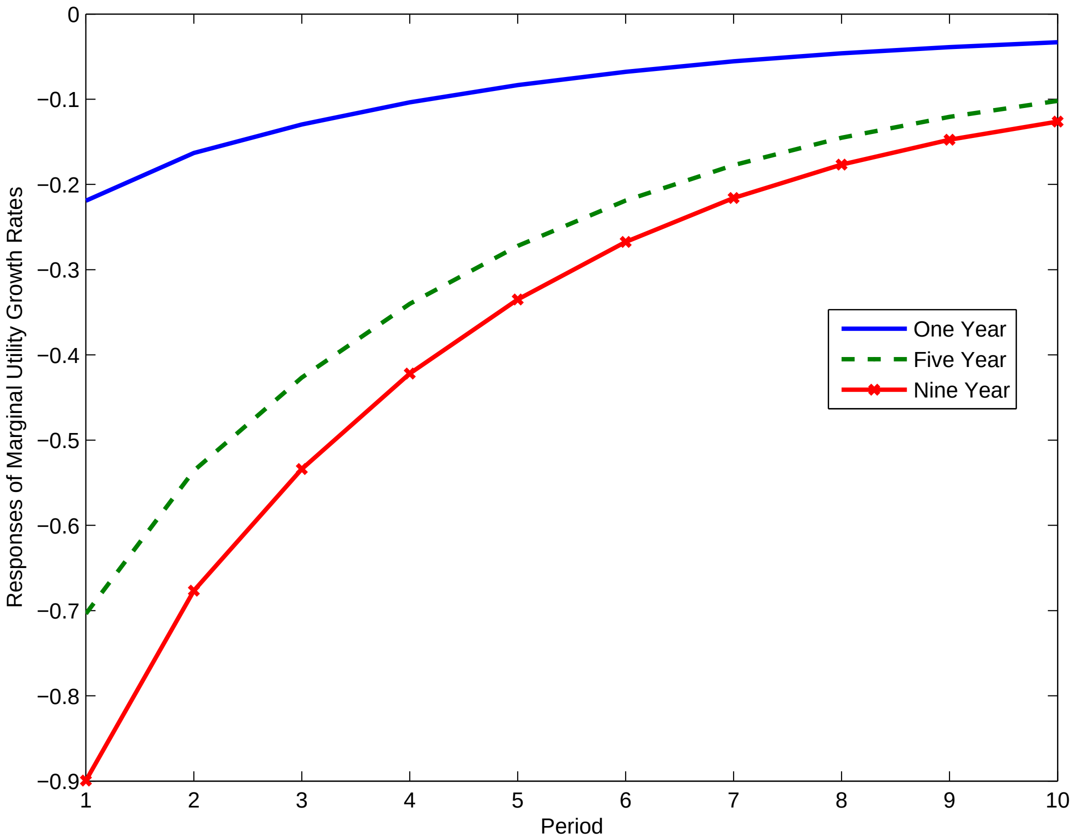


Notes: The figure shows the impulse responses of $E_t \frac{u_c(t+i)}{u_c(t)}$ for $i = 1, 5, 9$ to a one standard deviation increase in expenditures. The solid line represents a horizon of one year $i = 1$. The dashed line $i = 5$ and the crossed line $i = 9$. The economy is assumed to start from the steady state. The figure traces the responses over ten periods after the shock occurs in period 1.

For example, aggregate hours and consumption are not significantly affected. However, the unconditional sample volatility of the market value of debt and the volatility of the tax rate rise somewhat. Moreover, taxes gain slightly in persistence (even though persistence was high to begin with).

These implications confirm our earlier discussion that making government debt exclusively short term contributes towards increasing tax volatility and autocorrelation, consistent with the view that the gain from fiscal insurance for the government is now smaller. However, the changes in the properties of the statistics reported are not vast.

Figure 13: **Response of Expected Marginal Utility Growth Rates to the Technology Shock**



Notes: The figure shows the impulse responses of $E_t \frac{u_c(t+i)}{u_c(t)}$ for $i = 1, 5, 9$ to a one standard deviation drop in technology. The solid line represents a horizon of one year $i = 1$. The dashed line $i = 5$ and the crossed line $i = 9$. The economy is assumed to start from the steady state. The figure traces the responses over ten periods after the shock occurs in period 1.

5.3 Engineering Buybacks

Buybacks of long term government debt are not common in the US data. By and large the only episodes recorded over our sample period during which the US government removes from the secondary market long term debt before its maturity are the 2001 buybacks (see Greenwood and Vayanos (2010)), the quantitative easing rounds in 2009 and the so called operation twist in the 60s. Except for these episodes the most common practice is for debt once issued to be redeemed at maturity (see Marchesi (2004)).

In contrast to the empirical observations, theoretical models (such as the model of Angeletos (2002), Aiyagari et al. (2002)) typically make the assumption that in every period all government debt is bought in and new debt is issued. In the context of our model this amounts to forcing the government in every period t to purchase the long term bond it has issued in $t-1$. Evidently when long term debt is ten years in maturity, it is bought back in the next period as a nine year bond.

We investigate in this section the effects of buybacks reversing the assumption we have made so far that debt is not withdrawn from the market until it matures. To accomplish this we set up a slightly different version of our model, i.e. one that replaces the government budget constraint (21) with the following equation:

$$\sum_{i=\{1,N\}} b_t^i p_t^i = b_{t-1}^1 + \kappa_{t-1} b_{t-1}^N + p_t^{N-1} b_{t-1}^N + g_t - \tau_t z_t h_t \tag{44}$$

where $p_t^{N-1} = \beta^{N-1} E_t \frac{u_c(t+N-1)}{u_c(t)} + \sum_j \kappa_t \beta^j E_t \frac{u_c(t+j)}{u_c(t)}$ represent the price of a non zero coupon bond of maturity $N-1$. As before, we keep the structure that the only two bonds which are available to the market are a one year and a ten year bond. We therefore set $N = 10$.

Since all remaining equations are essentially the same as the equations derived in section 4, we refer the reader to that section for brevity.

5.3.1 Estimating the debt management rule under buybacks

To show the behavior of the economy in the case of buybacks of government debt we have to re-estimate equation (16). This is so because under buybacks the maturity structure in our model is no longer given by the following expression:

$$MAT_t = \frac{1}{MV_t}(b_t^N (p_t^N N + \sum_{j\in\{2,4,\ldots N\}} j p_t^j \kappa_t))$$
$$+ b_{t-1}^N (p_t^{N-1}(N-1) + \sum_{j\in\{1,2,\ldots N-1\}} j p_t^j \kappa_{t-1}) + \ldots. + b_{t-N+1}^N p_t^1 (1 + \kappa_{t-N+1}) + b_t^1 p_t^1)$$

Rather since bonds are not redeemed at maturity the appropriate expression for the maturity structure is the following:

$$MAT_t = \frac{1}{MV_t}(b_t^N (p_t^N N + \sum_{j\in\{1,2,\ldots N\}} j p_t^j \kappa_t)) + b_t^1 p_t^1) \tag{45}$$

Note that at any date the stock of debt is simply the issuance in that period (i.e. there are no past issuances which matter for the value of debt and the maturity structure). Equation (45) can be further rearranged into:

$$MAT_t = s_t^1 \omega_t + (1 - s_t^1)\omega_t \xi_t$$

With this notation, and since all debt is bought in at every period, the term ω_t (ratio of issuance to the market value of debt) is constant and equal to one.

Utilizing the linearization procedure, we employed previously, we can arrive to the following expression for the maturity structure:

$$MAT_t = MAT + (s_t^1 - s^1) - (s_t^1 - s^1)\xi + (1 - s^1)(\xi_t - \xi)$$

To estimate the above equation we utilize the procedure outlined in section 3. Our estimates of the sharing rule are as follows: We obtain a value of the constant equal to 0.0205, a first order autocorrelation coefficient of 0.9555 and a value for ω_2 equal to -0.0167. [21] These values are quite different from the analogous values we obtained in section 3. This, however, is not surprising since as we said the issuance in this section coincides with the market value. We get a smaller coefficient for ω_2 but with this estimate we can match the response of the maturity structure to the debt to GDP ratio. Moreover, the autocorrelation coefficient gives a higher value since the market value of government debt in the data exhibits higher serial correlation, and also the share of short term debt in the market value is more persistent than the analogous share in the issuance.

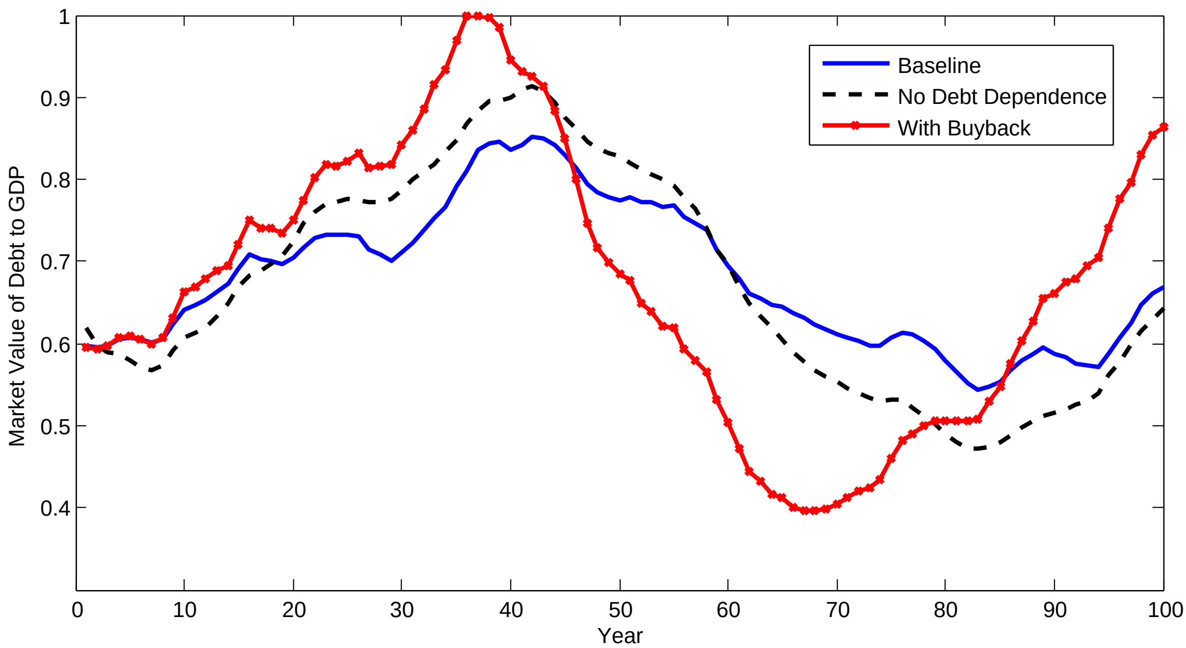
5.3.2 Quantitative Results

In Figure 14 we illustrate the behavior of the market value of government debt. The solid line represents the baseline model, the dashed line the 'No Debt Dependence' model of the previous paragraph, and the crossed line shows the buyback model of this section. As is shown in the figure, assuming buybacks change considerably the behavior of the debt

[21] The standard deviation of the error term was found to be equal to 0.0407.

aggregate. Under the specification of the model of this section, the market value fluctuates over a wider range, however it also displays less persistence relative to the models studied in the previous sections.

Figure 14: **Market Value of Debt / GDP**



Notes: The figure shows the behavior of the tax revenue. The solid line represents the baseline model (blue). The dashed line represents the 'zero dependence' model (green). The starred line represents the 'no buyback' model (red).

This difference in the behavior cannot be attributed to a difference in the average maturity structure between the two models. Since our estimates of the sharing rule were targeted to fit the properties of the average maturity of debt in the US data, our simulations in the models deliver roughly the same value for this quantity over time. However, even though the average maturity is similar there is a stark difference in the distribution of maturities across the two models. To be more precise, under no buyback, long term debt which has been issued in the past works its way through the maturity structure. For example, debt which is close to maturity is effectively short term whereas debt which was issued say five years ago is effectively today five year maturity debt. This gives to the no buyback model a much richer distribution of maturities at any point in time than in the buyback model. Under buybacks, bond quantities in each period are concentrated to one year and ten year maturities.

These differences in the maturity structure may explain the different responses of the market value we see in figure 14. Given the above remarks, it is obvious that under the buyback model there is a lot more one year debt outstanding. As we discussed in section 2.2, one year debt gives no fiscal insurance to the government. Therefore, the market value of government debt may be more volatile and responsive to shocks should fiscal insurance be limited in the presence of short term debt. This may explain why the market value of government debt in the figure swings over a wider range of values in the buyback model. It is also clear that tax rates follow this behavior.

5.3.3 Moments under buyback

Rows 3 and 6 columns of Table 2 show the key moments from the buyback model. Notice that buybacks reduce considerably the volatility of aggregate hours, increase somewhat the volatility of consumption, and reduce the volatility of the market value of debt, tax rates and tax revenues. These implications may seem counter-intuitive in light of our previous arguments that buybacks produce less fiscal hedging (and therefore we would anticipate the variance of debt to increase). However we also showed that the market value of debt is less persistent and therefore its overall unconditional volatility is smaller. Since debt is the only source of variation in taxes we also get that tax rates are less volatile.

6 Conclusion

This thesis considers the effects of alternative debt management strategies on the behavior of the market value of government debt as well as on the behavior of important economic variables, such as hours, consumption and taxes. We study these effects through the lens of a structural economic model featuring an optimizing private sector and a government which sets distortionary taxes and chooses the debt portfolio to finance deficits.

To calibrate the model to the data, we estimate the debt management rule from the data. In particular, we estimate the share of short term debt (one year debt in the model) over the total issuance of debt in any year from 1955 to 2011. This sharing rule relates the share in a given year to its first order lag and the debt to GDP ratio.

To simplify, we assume that the debt management authority issues debt in two maturities: A one year and a ten year bond. This formulation follows the related literature on optimal debt management. In order to map this model to the data, we utilize a first order approximation of the average maturity of government debt. This enables us to drop the history of past issuances and to simplify the econometric model. By estimating the sharing rule, we attempt to match the average maturity in the historical observations. We find a significant (negative) relationship between the share of short term bonds in the portfolio of the government and the debt level. We also find that the share exhibits strong persistence over time.

With these parametric estimates we calibrate our structural model. After analyzing the behavior of the economy under the benchmark calibration of the model (i.e. the one which captures the current policies in the US), we consider how changes in the debt management rule, may affect economic outcomes. We show that in the case where the government issues only short maturity debt (as opposed to targeting a portfolio which consist of both short and long bonds) and in the case where it buys back its debt in every period, there are significant effects on the dynamics of the market value to GDP ratio, however there are more moderate effects on consumption and hours. To motivate these findings we appeal to the theory of fiscal insurance, which suggests that long maturity government bonds have a hedging value to the intertemporal budget.

The conclusion we draw is that, to the extend that governments are concerned about high debt levels, the choice of the debt management regime is important as it affects the dynamics of the debt aggregate.

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7 Data Appendix

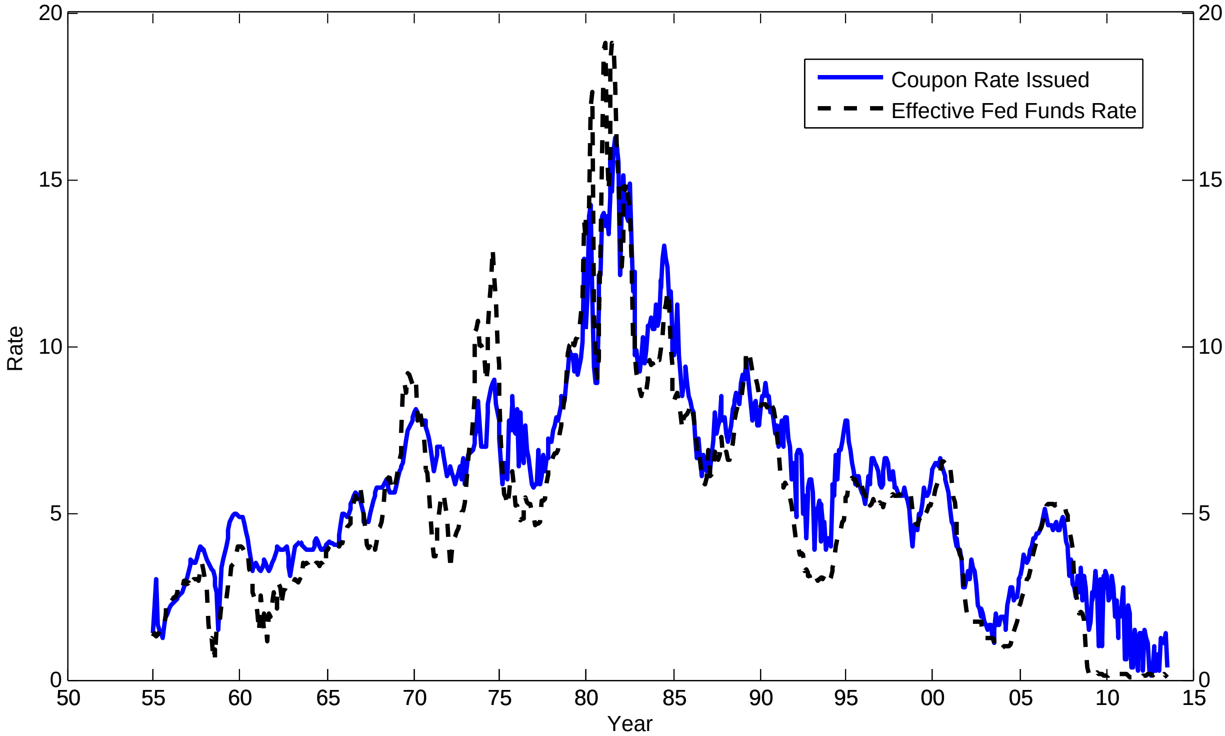
Data on US Treasuries was obtained from the CRSP US Treasury Database and is comprised of all types of bonds, including callable securities and TIPS. Observations were available at a monthly frequency and were taken from the period 1955-2011. Date variables of particular interest for this study included the quote date, the date of the first coupon and the maturity date. Amounts outstanding of the bonds were usually available although missing for certain observations. Gaps in amounts outstanding were filled with preceding observations when available for the same bond or future observations if no preceding data points exist. Bonds were stripped and individual coupons, now strips, were given distinct maturity dates, face values and market values. Market values for strips were priced using the yield-to-maturity data (available from the database as a one-day rate) for the principal bond from which they were separated. The parent bond's price outstanding was adjusted by subtracting the present value of its coupons.

To give an example, a 5-year treasury bond yielding a 5 percent coupon would have ten strips maturing six months from one another with the last one paid along with the principal at maturity, and each with a $2.5 face value for every one-hundred dollars of face value of the original bond. Each strip would be turned into a zero-coupon bond with a $2.5 face value, discounted appropriately using its new maturity date. Each consecutive strip would mature six months from the previous one. Should the original bond have been trading near par, removing the strips would turn it into a discount bond.

As noted in the paper, while coupon bonds are usually floated near par value (i.e. at a market price equal to the principal amount), it is not always the case. This is because the coupon size does not always equal the current discount rate at a given moment in time, although the Federal Reserve has historically issued coupons of sizes similar to the present Federal Funds Rate which means that newly issued bonds tend to be sold at or near par-value (Figure 15).

The time series of average maturity of debt found in Figures 1 and 2 was calculated using outstanding bond data (including strips) in a period weighted by their outstanding amount. Share of short term debt in Figure 3 was calculated by taking amounts outstand-

Figure 15: **Coupon Rate of Issued Coupon Bonds vs. Effective Federal Funds Rate**



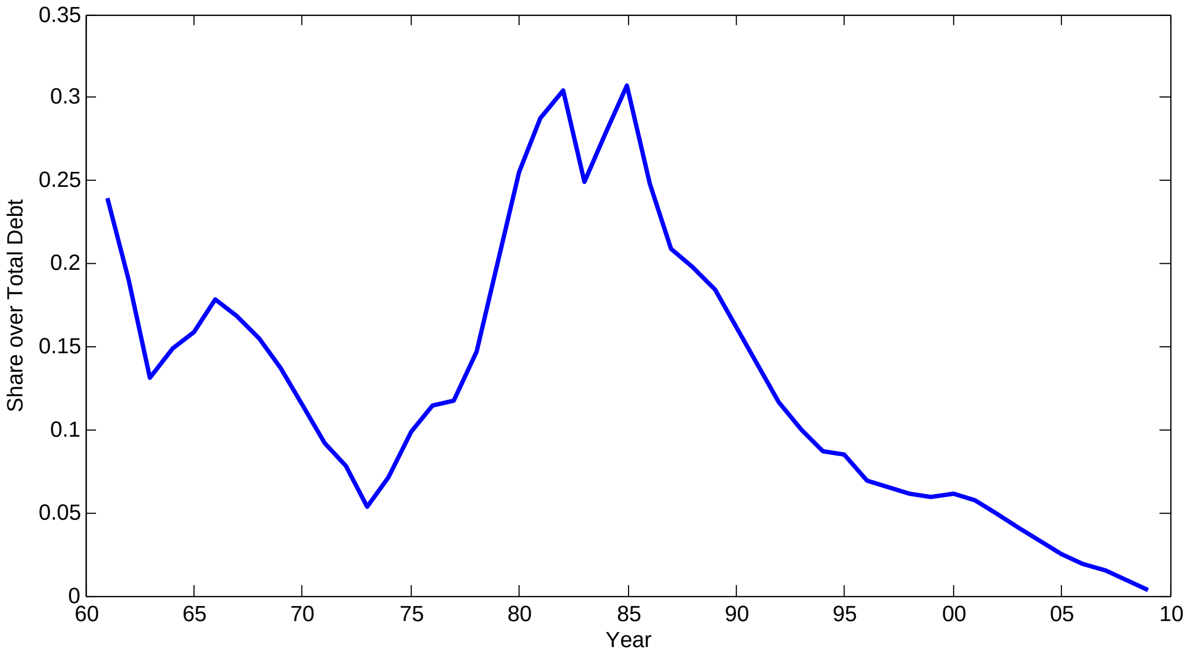
Notes: The Figure plots the effective Federal Funds Rate (taken from the St.Louis Fed FRED database) and coupons of bond issues over the period 1955-2013. The data are monthly observations with a small number of interpolated observations when coupons were missing for certain months.

ing of debt maturing withing 12 months over total debt outstanding for a given period, likewise for the maturity buckets in Figures 4.

7.1 Callable bonds

A callable bond, also known as a redeemable bond, is a bond with an embedded option which allows the issuer the right to redeem the principal of the bond (at par) from its holders at specific dates before maturity. These call dates are the coupon dates of the bond and are only active past a certain period (e.g.: the last five years of a 10-year treasury bonds life). The difference between the market prices of a callable treasury and a non-callable equivalent should reflect this optionality, and thus a callable bond should in theory never sell at a higher price than a non-callable equivalent (note that a non-callable bonds face value outstanding is only redeemed by the government at maturity). Given that the CRSP US Database includes the yield-to-maturity (YTM) data throughout all of its monthly observations, this optionality price differential is already reflected in the data when doing analysis with the market value of bonds. It is worth noting that callable bonds have been phased out by the Treasury since their peak in 1984 (see Figure 16).

Figure 16: **Share of callable Treasuries**



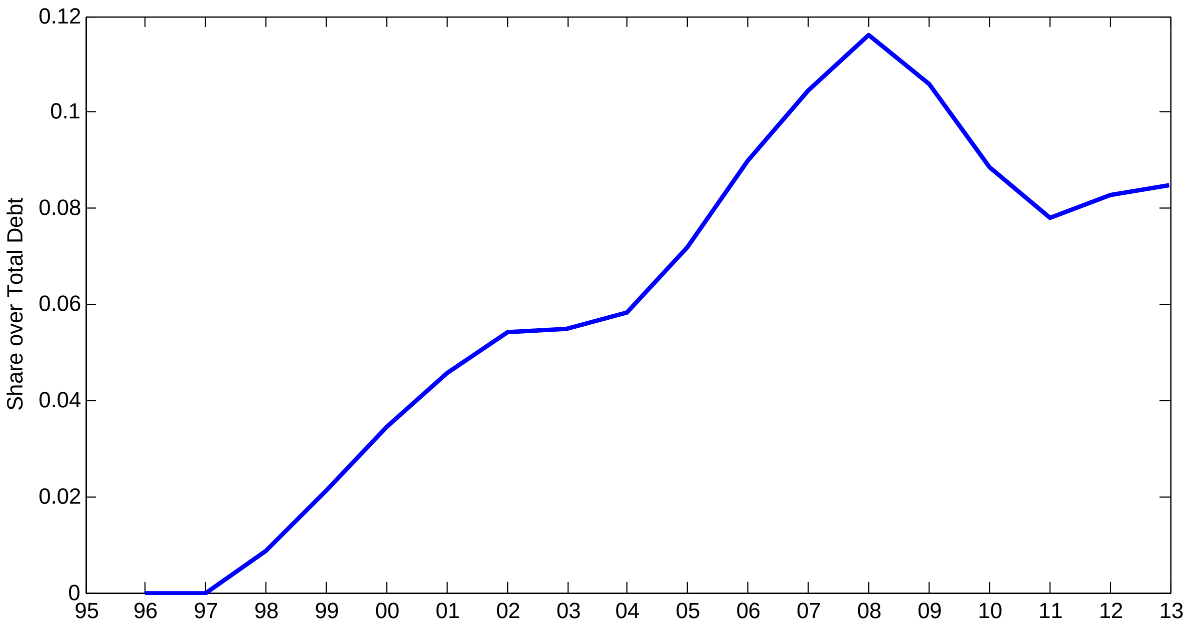
Notes: The Figure plots the share of callable debt over total debt outstanding in the US over the period 1961-2008. The data are annual observations (time aggregated from monthly data extracted from the CRSP).

7.2 Inflation-indexed bonds

Treasury Inflation-Protected Securities (TIPS) are Treasury bonds with principal amounts (and their associated coupon payments, which are a percentage of that principal) that adjust with the Consumer Price Index (CPI). These securities are sought after for their ability to protect a bondholder from inflation which erodes real returns. According to the Treasurys official website, the first TIPS was floated in 1997. TIPS have been offered in

5-year, 10-year and 30-year maturities and their amounts have steadily grown over the years (see Figure 17). The exact mechanic behind the adjustment methodology uses an index ratio which is calculated by dividing the recently observed reference CPI by the CPI calculated right before the TIPS was issued. The par-value principal amounts are then multiplied by these index ratios to obtain the adjusted principal amounts of a TIPS at a given date as well as its interest payment which is based on the bond's coupon rate. The reference CPI used to calculate this index ratio is actually lagged by two months since the CPI is calculated using past data and never in real time. Furthermore, daily reference CPIs are linearly interpolated between the monthly CPI data points. We have adjusted the interest payments in the data to reflect these changing principal amounts.

Figure 17: **Share of TIPS in the US**



Notes: The Figure plots the share of real debt over total debt outstanding in the US over the period 1995-2011 (TIPS were introduced in 1997). The data are annual observations (time aggregated from monthly data extracted from the CRSP).

Nominal outstanding amounts were adjusted for inflation using the Consumer Price Index for All Urban Consumers (all items) available from FRED Economic Data (St.

Louis Fed website). The graph of the short-term share of government debt matches results by Greenwood et al. (2013) with discrepancies apparent after 1997 explained by the inclusion of TIPS.